OPTIMIZATION CRITERIA FOR DESIGN OF TUNED MASS DAMPERS INCLUDING SOIL–STRUCTURE INTERACTION EFFECT

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ABSTRACT

Many researches have focused on the optimal design of tuned mass damper (TMD) system without the effect of soil–structure interaction (SSI), so that ignoring the effect of SSI may lead to an undesirable and unrealistic design of TMD. Furthermore, many optimization criteria have been proposed for the optimal design of the TMD system. Hence, the main aim of this study is to compare different optimization criteria for the optimal design of the TMD system considering the effects of SSI in a high–rise building. To achieve this purpose, the optimal TMD for a 40–storey shear building is firstly evaluated by expressing the objective functions in terms of the reduction of structural responses (including the displacement and acceleration) and the limitation of the scaled stroke of TMD. Then, the best optimization criterion is selected, which leads to the best performance for the vibration control of the structure. In this study, the whale optimization algorithm (WOA) is employed to optimize the parameters of the TMD system. The numerical results show that the soil type and selected objective function efficiently affect the optimal design of the TMD system.

Keywords: tuned mass damper; soil–structure interaction; optimization criteria; optimal design; whale optimization algorithm; transfer function.

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1. INTRODUCTION

When a high–rise (or tall) building is excited by a severe earthquake, the safety and serviceability of the structure can be reduced or destroyed. In order to protect high–rise buildings subjected to earthquake loading, several strategies of vibration control have been...
proposed and developed. Tuned mass damper (TMD) systems have been introduced as one of the passive control devices. The system consists of a mass–spring–dashpot system installed on a storey of a primary structure [1]. Finding the optimum position for installing the TMD system on the structure and also the optimal values for the parameters of the TMD system (mass, stiffness and damping) have been considered as the most important problems and have been widely investigated in the seismic design of controlled building [2–7].

For a high–rise building located on a soft soil, its seismic responses are significantly different from those with a fixed base or very stiff soil. In other words, the soil–structure interaction (SSI) phenomenon affects the seismic responses of the high–rise building founded on soft soil. Thus, considering the effects of SSI is important in the seismic design and vibration control of structures [8–9]. A number of researches have investigated the SSI effect on the vibration control effectiveness of structures [10–14].

Den Hartog [15] firstly proposed the optimum parameters of the TMD system for an undamped single degree of freedom (SDOF) system subjected to a harmonic loading. The optimum parameters of the TMD system for a damped SDOF system investigated by several researchers. With the advent of numerical methods and metaheuristic optimization algorithms, the optimum parameters of the TMD system were widely computed for a damped SDOF or multi degree of freedom (MDOF) systems subjected to the arbitrary lateral loading such as the wind and earthquake loadings [16–30]. The optimum design of the TMD system considering the SSI effects has been proposed and developed by Farshidianfar and Soheili [31–32]. Rahai et al. have recently demonstrated that the formulation of a high–rise building controlled by the TMD system and considering the SSI effects by Farshidianfar and Soheili [31–32] was not accurate enough [33]. Recently, Bekdaş and Nigdeli [2] have proposed an optimization approach for the optimum design of TMDs considering SSI effects.

In the optimal design of the TMD system, the choice of a suitable objective function has been considered as an important challenge. Hence, this study compares different optimization criteria for the optimal design of the TMD system considering the effects of SSI in a high–rise building. To achieve this purpose, the comparison between the optimal design of a TMD for a 40–storey shear building is performed which is obtained considering a number of distinct TMD optimization criteria. The criteria are based on the minimization of the displacement and acceleration for the structural responses subjected to a design constraint including the scaled stroke of TMD. In this study, the whale optimization algorithm (WOA) is used to optimize the parameters of the TMD system. The numerical results show that the soil type and the choice of a suitable objective function efficiently affect the optimal design of the TMD system.

2. STATEMENT OF OPTIMIZATION PROBLEM FOR A TMD SYSTEM

The main aim of this paper is to optimize the design of a TMD system for a high–rise building considering the SSI effect. In fact, the optimal parameters of the TMD system (including damping, stiffness and mass) are determined in the framework of an optimization problem. In this paper, five different objective functions as the optimization criteria are considered. For each of them, the optimum parameters of the TMD system are firstly computed. Finally the seismic performance of the building corresponding to each of the
optimized TMD system is compared in other to find the best objective function. Thus, the optimal design of a TMD system for a high-rise building considering the SSI effect can be formulated as:

\[
\begin{align*}
\text{Find:} & \quad M_{\text{TMD}}, K_{\text{TMD}}, C_{\text{TMD}} \\
\text{Minimize:} & \quad \text{OF}(M_{\text{TMD}}, K_{\text{TMD}}, C_{\text{TMD}}) \\
\text{Subjected to:} & \quad M_{\text{TMD}}^{\text{min}} \leq M_{\text{TMD}} \leq M_{\text{TMD}}^{\text{max}} \\
& \quad K_{\text{TMD}}^{\text{min}} \leq K_{\text{TMD}} \leq K_{\text{TMD}}^{\text{max}} \\
& \quad C_{\text{TMD}}^{\text{min}} \leq C_{\text{TMD}} \leq C_{\text{TMD}}^{\text{max}} \\
& \quad \frac{\max|\dot{x}_{\text{TMD}}(t) - x_{\text{Roof}}(t)|_{\text{with TMD}}}{\max|x_{\text{Roof}}(t)|_{\text{without TMD}}} \leq s_{\text{f, max}}
\end{align*}
\]

where \(M_{\text{TMD}}, K_{\text{TMD}}\) and \(C_{\text{TMD}}\) are mass, stiffness and damper of TMD, respectively. \(M_{\text{TMD}}^{\text{min}}\) and \(M_{\text{TMD}}^{\text{max}}\) are the lower and upper bounds of the TMD mass, respectively. \(K_{\text{TMD}}^{\text{min}}\) and \(K_{\text{TMD}}^{\text{max}}\) are the lower and upper bounds of the TMD spring constant, respectively. \(C_{\text{TMD}}^{\text{min}}\) and \(C_{\text{TMD}}^{\text{max}}\) are the lower and upper bounds of the TMD damping constant, respectively. The design constraint defined as Eq. (1) is considered as the stroke capacity of TMD and is limited to a user-defined value, \(s_{\text{f, max}}\).

The first objective function is proposed in this study and is defined by a combination of the maximum values of acceleration transfer function and displacement of the roof story subjected to an earthquake loading and is expressed as:

\[
\text{OF}_{1} = \frac{\max|\text{TF}_{\text{ACC, Roof}}|_{\text{with TMD}}}{\max|\text{TF}_{\text{ACC, Roof}}|_{\text{without TMD}}} + \frac{\max|x_{\text{Roof}}(t)|_{\text{with TMD}}}{\max|x_{\text{Roof}}(t)|_{\text{without TMD}}}
\]

where \(\text{TF}_{\text{ACC, Roof}}\) is the transfer function of the acceleration for the roof story. The transfer function is defined by the ratio of the Laplace transformations of the acceleration and ground acceleration in decibels (dB).

The second objective function proposed by Yazdi et al. [3] is represented by a combination of the maximum values of acceleration and displacement transfer function for the top story (roof) subjected to an earthquake loading and is defined as:

\[
\text{OF}_{2} = \frac{\max|\text{TF}_{\text{ACC, Roof}}|_{\text{with TMD}}}{\max|\text{TF}_{\text{ACC, Roof}}|_{\text{without TMD}}} + \frac{\max|\text{TF}_{\text{Dis, Roof}}|_{\text{with TMD}}}{\max|\text{TF}_{\text{Dis, Roof}}|_{\text{without TMD}}}
\]

where \(\text{TF}_{\text{Dis, Roof}}\) is the transfer function of the displacement for the roof story.

The maximum of the roof displacement for the controlled structure is considered as the third objective function and in the following form [2]:

\[
\text{OF}_{3} = \frac{\max|x_{\text{Roof}}(t)|_{\text{with TMD}}}{\max|x_{\text{Roof}}(t)|_{\text{without TMD}}}
\]
In this study, the combination of the maximum values of displacement transfer function and displacement for the roof story subjected to an earthquake loading is proposed as the fourth objective function. The objective function is given by:

$$\text{OF 4} = \max\left[ \frac{\text{TF}_{\text{Des. Roof}} \text{ with TMD}}{\text{TF}_{\text{Des. Roof}} \text{ without TMD}} \right] + \max\left[ \frac{x_{\text{Roof}}(t) \text{ with TMD}}{x_{\text{Roof}}(t) \text{ without TMD}} \right]$$

Finally, the root–mean–square (RMS) of the roof displacement for the controlled structure is considered as the fifth objective function. This objective function is defined as [4]:

$$\text{OF 5} = \sqrt{\frac{\sum_{i=1}^{T} x_{\text{Roof}}(t)}{T}}$$

where $T_{\text{max}}$ is the total number of time steps for a ground acceleration.

### 3. MATHEMATIC EQUATIONS OF A STRUCTURE WITH SSI

#### 3.1 A structure including a TMD system and SSI

In this section, the equations of motion of a controlled high–rise structure considering the SSI effect are represented, which are obtained using the Lagrangian method [34]. Fig. 1 shows a $N$–storey shear building with a TMD and its subsoil model. $M_i$, $C_i$, $K_i$, and $x_i$ represent the mass, damping, stiffness and the displacement for the $i$th storey, respectively. The parameters of soil and foundation contain by the mass of foundation ($M_0$), the mass moment of inertia of foundation ($I_0$), the damping of the swaying dashpot ($C_s$), the damping of the rocking dashpots ($C_r$), the stiffness of swaying motion ($K_s$) and the stiffness of the soil rocking motion ($K_r$).

The main form of the equations of motion for a $N$–storey shear building structure including the SSI effect and a TMD is given in Eq. (7) as follows [35]:

$$[\mathbf{M}][\ddot{x}(t)] + [\mathbf{C}][\dot{x}(t)] + [\mathbf{K}][x(t)] = -[\mathbf{m}^*][f] \ddot{g}(t)$$

In Eq. (7), $[\mathbf{M}]$, $[\mathbf{C}]$ and $[\mathbf{K}]$ show the mass, damping, and stiffness of the structure including the TMD system and the SSI effect, respectively. The $[\dot{x}(t)]$, $[x(t)]$ and $[\ddot{x}(t)]$ represent the acceleration, velocity and displacement vectors, respectively. Also, $[\mathbf{m}^*]$, $[f]$ and $\ddot{g}(t)$ indicate acceleration mass matrix for earthquake, influence vector and the
OPTIMIZATION CRITERIA FOR DESIGN OF TUNED MASS DAMPERS INCLUDING...

earthquake acceleration, respectively. In Eq. (7), the mass matrix can be computed as follows:

\[
\begin{bmatrix}
\mathbf{M}_s\end{bmatrix}_{N \times N} = \begin{bmatrix}
M_1 & 0 & 0 & 0 & 0 \\
M_2 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & M_{N-1} & 0 \\
0 & 0 & 0 & 0 & M_N
\end{bmatrix}_{\text{sym}}
\]

\[
\begin{bmatrix}
\mathbf{M}_d\end{bmatrix}_{N \times 1} = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
0
\end{bmatrix}_{N \times 1}
\]

\[
\begin{bmatrix}
\mathbf{M}_T\end{bmatrix}_{N \times 1} = \begin{bmatrix}
M_1Z_1 \\
M_2Z_2 \\
\vdots \\
M_{N-1}Z_{N-1} \\
M_NZ_N
\end{bmatrix}
\]

\[
\mathbf{M} = \begin{bmatrix}
\mathbf{M}_s & \mathbf{M}_T \\
\mathbf{M}_d & \mathbf{M}_T
\end{bmatrix}
\]

Figure 1. Model of a \( N \)-storey shear building structure including a TMD system and the SSI effect

\[
\begin{bmatrix}
\mathbf{M} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{M}_T & \mathbf{M}_T & \mathbf{M}_T & \mathbf{M}_T & \mathbf{M}_T \\
\mathbf{M}_d & \mathbf{M}_T & \mathbf{M}_T & \mathbf{M}_T & \mathbf{M}_T \\
\mathbf{M}_T & \mathbf{M}_d & \mathbf{M}_T & \mathbf{M}_T & \mathbf{M}_T \\
\mathbf{M}_T & \mathbf{M}_T & \mathbf{M}_d & \mathbf{M}_T & \mathbf{M}_T \\
\mathbf{M}_T & \mathbf{M}_T & \mathbf{M}_T & \mathbf{M}_d & \mathbf{M}_T \\
\mathbf{M}_T & \mathbf{M}_T & \mathbf{M}_T & \mathbf{M}_T & \mathbf{M}_d
\end{bmatrix}
\]

\[
\begin{bmatrix}
\mathbf{z}_N \\
\mathbf{z}_N \\
\vdots \\
\mathbf{z}_N \\
\mathbf{z}_N \\
\vdots \\
\mathbf{z}_N
\end{bmatrix}
\]

\[
\begin{bmatrix}
\mathbf{m}_N \\
\mathbf{m}_N \\
\vdots \\
\mathbf{m}_N
\end{bmatrix}
\]

\[
\begin{bmatrix}
\mathbf{a}_N \\
\mathbf{a}_N \\
\vdots \\
\mathbf{a}_N
\end{bmatrix}
\]

\[
\mathbf{a}_N = \begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix}
\]

where

\[
\mathbf{M}_0 + \sum_{j=1}^{N} M_j + M_{TMD}
\]

\[
I_0 + \sum_{j=1}^{N} (j + M_j Z_j^2) + M_{TMD} Z_N^2
\]

\[
(8)
\]

The damping matrix is also defined as follows:

\[
\begin{bmatrix}
\mathbf{M}_s & \mathbf{M}_T \\
\mathbf{M}_d & \mathbf{M}_T
\end{bmatrix}
\]

\[
(9)
\]
\[ [C]_{(N+3)\times(N+3)} = \begin{bmatrix} \mathbf{C}_{ST} & \{0\}_{(N+1)\times1} & \{0\}_{(N+1)\times1} \\ \text{sym} & C_s & 0 \\ \text{sym} & C_r & \end{bmatrix} \] (10)

where

\[ \mathbf{C}_{ST}_{(N+1)\times(N+1)} = \begin{bmatrix} C_1 + C_2 & -C_2 & 0 & 0 & 0 \\ -C_2 & C_2 + C_3 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \text{sym} & C_N + C_{TM} & -C_{TM} & -C_{TM} & C_{TM} \end{bmatrix} \] (11)

In addition, the stiffness matrix can be computed using the following equation:

\[ [K]_{(N+3)\times(N+3)} = \begin{bmatrix} \mathbf{K}_{st} & \{0\}_{(N+1)\times1} & \{0\}_{(N+1)\times1} \\ \text{sym} & K_s & 0 \\ \text{sym} & K_r & \end{bmatrix} \] (12)

where

\[ \mathbf{K}_{st}_{(N+1)\times(N+1)} = \begin{bmatrix} K_1 + K_2 & -K_2 & 0 & 0 & 0 \\ -K_2 & K_2 + K_3 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \text{sym} & K_N + K_{TM} & -K_{TM} & -K_{TM} & K_{TM} \end{bmatrix} \] (13)

Finally, the acceleration mass matrix is computed as follows:

\[ [\mathbf{m}^*] = \begin{bmatrix} \mathbf{M}_s_{N\times N} & \{0\}_{N\times1} & \{0\}_{N\times1} & \{0\}_{N\times1} \\ 0 & M_{TM} & 0 & 0 \\ 0 & 0 & M_0 + \sum_{j=1}^{\frac{N}{2}} M_j + M_{TM} & 0 \\ 0 & 0 & \sum_{j=1}^{\frac{N}{2}} M_j Z_j + M_{TM} Z_N & 0 \end{bmatrix} \] (14)

The damping matrix can be computed using the Rayleigh method [35], as follows:

\[ [\mathbf{C}_s]_{N\times N} = A_0 [\mathbf{M}_s]_{N\times N} + A_1 [\mathbf{K}_s]_{N\times N} \] (15)
where $A_0$ and $A_1$ are the Rayleigh damping ratios that can be computed by Eq. (12):

$$A_0 = \frac{2\xi_0 \omega_0}{(\omega_0 + \omega_j)}$$

$$A_1 = \frac{2\xi_1 \omega_j}{(\omega_j + \omega_j)}$$

(16)

where $\xi$ is the damping ratio. $\omega_i$ and $\omega_j$ are the natural frequency of the structure for the $i^{th}$ and $j^{th}$ modes, respectively. Furthermore, in Eq. (7), the vector $\{x(t)\}$ includes the displacement of the structure, TMD system and the displacement and rotation of the foundation, which is defined as follows:

$$\{x(t)\} = \{x_1(t), x_2(t), ..., x_N(t), x_{TMD}(t), x_{0}(t), \theta_{0}(t)\}^T$$

(17)

Furthermore, the displacement of the top (roof) story is determined as:

$$x_{Roof}(t) = x_0(t) + \theta_0(t) \times Z_N + x_N(t)$$

(18)

3.2 A structure considering the effect of SSI

The equations of motion for a structure considering the SSI effect are defined as Eq. (7). The mass matrix of this system can be computed as follows:

$$\begin{bmatrix} M_0 + \sum_{j=1}^{N} M_j \\ M_{sym} \\ I_0 + \sum_{j=1}^{N} (I_j + M_j Z_j^2) \end{bmatrix}_{(N+2) \times (N+2)}$$

(19)

The damping matrix is also defined as follows:

$$\begin{bmatrix} C_s(N) \times (N) & \{0\}_{(N) \times 1} \\ \{0\}_{(N) \times 1} & C_r \end{bmatrix}$$

(20)

In addition, the stiffness matrix can be computed using the following equation:
\[ [\mathbf{K}]_{(N+2)\times(N+2)} = \begin{bmatrix} [\mathbf{K}_s]_{(N)\times(N)} & [\mathbf{0}]_{(N)\times1} & [\mathbf{0}]_{(N)\times1} \\
\mathbf{0} & \mathbf{K}_r & \mathbf{0} \\
sym & \mathbf{K}_r & \mathbf{0} \end{bmatrix} \] (21)

where

\[ [\mathbf{K}_s]_{(N)\times(N)} = \begin{bmatrix} K_1 + K_2 & -K_2 & 0 & 0 & 0 \\
-K_2 & K_2 + K_3 & 0 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & K_{N-1} + K_N & -K_N \\
0 & 0 & 0 & -K_N & K_N \end{bmatrix} \] (22)

Finally, the acceleration mass matrix is computed as follows:

\[ [\mathbf{m}]^n = \begin{bmatrix} [\mathbf{M}_s]_{N\timesN} & [\mathbf{0}]_{N\times1} & [\mathbf{0}]_{N\times1} \\
[\mathbf{0}]_{1\timesN} & M_0 + \sum_{j=1}^{N} M_j & 0 \\
[\mathbf{0}]_{1\timesN} & \sum_{j=1}^{N} M_j Z_j & 0 \end{bmatrix} \] (23)

Furthermore, in Eq. (3), the vector \([x(t)]\) includes the displacement of the structure, and displacement and rotation of the foundation that is defined as follows:

\[ \mathbf{X}(t) = [x_1(t), x_2(t), \ldots, x_N(t), \theta_0(t)]^T \] (24)

In this study, the Newmark method [35] is used for solving the motion equations defined in Eq. (7).

4. WHALE OPTIMIZATION ALGORITHM

Whale optimization algorithm (WOA) proposed by Mirjalili and Lewis [36] is a novel meta–heuristic algorithm which mimics the social behavior of humpback whales. This algorithm is implemented based on the spiral bubble–net feeding maneuver. In order to update the position of the whales during optimization procedure, the shrinking encircling mechanism and the spiral bubble–net feeding maneuver is used. In the basic WOA, it is assumed that the current best candidate solution can be considered as the optimum or is close to the optimum. Hence, the other search agents will update their positions towards the best search agent [36].

The WOA includes two phases (i.e exploitation and exploration phase) and transits between exploration and exploitation phase smoothly. The transition is implemented by the
OPTIMIZATION CRITERIA FOR DESIGN OF TUNED MASS DAMPERS INCLUDING...

variation of $A$ vector’s value. $A$ vector’s value is decreased during iterations, half of iterations are assigned to exploration phase when $|A| \geq 1$ and the other half is dedicated to exploitation when $|A| < 1$. Here, the sign $||$ indicates the absolute value. The vector $A$ is computed as follow:

$$A = 2ar - a$$

(25)

where $a$ is linearly decreased from 2 to 0 over the course of iterations and $r$ is a random vector in $[0,1]$.

4.1 Bubble–net attacking method (exploitation phase)

For modeling the bubble–net behavior of humpback whales, two approaches including shrinking encircling mechanism and spiral updating position were proposed [36]. Since the humpback whales swim around the prey within a shrinking circle and along a spiral–shaped path simultaneously, it is assumed in WOA that there is a probability of 50% to choose between these two behaviors. The shrinking encircling mechanism is modeled as follow:

$$\bar{C} = 2r$$

(26)

$$D = |C\bar{X}(l) - \bar{X}(l)|$$

(27)

$$\bar{X}(l+1) = \bar{X}^*(l) - AD$$

(28)

where $\bar{X}$ is the position of whales, respectively. $\bar{X}^*$ is the position vector of the best solution obtained so far. Furthermore, the spiral–shape movement of whales is simulated as following formulas:

$$D' = |\bar{X}^* - X(l)|$$

(29)

$$\bar{X}(l+1) = D'e^{bp}\cos(2\pi p) + \bar{X}^*$$

(30)

where $b$ is a constant that define the spiral shape of movement. $p$ is a random number in $[-1,1]$.

4.2 Search for prey (exploration phase)

In the exploration phase of WOA, the position of a whale is updated based on the a randomly chosen whale instead of the best search whale. Thus, the new position of whales is obtained as:

$$D = |C\bar{X}^*(l) - \bar{X}(l)|$$

(31)

$$\bar{X}(l+1) = \bar{X}^*(l) - AD$$

(32)
In this paper, the optimal design of a TMD system for a 40–storey shear building studied by Farshidianfar and Soheili [31–32] is investigated. The properties of the structure are shown in Table 1. While, each story has the same mass, height and moment of inertia, the stiffness of the structure linearly decreases when \( Z \) distances increase.

**Table 1: The parameters of the studied structure [31–32]**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of each storey ((m))</td>
<td>4</td>
</tr>
<tr>
<td>Mass of each storey ( (N\cdotsec^2/m) )</td>
<td>(9.8\times10^5)</td>
</tr>
<tr>
<td>Inertia moment of each storey ( (kg\cdotm^2) )</td>
<td>(1.31\times10^8)</td>
</tr>
<tr>
<td>Stiffness of stories ( (N/m) )</td>
<td>(K_1=2.13\times10^9 - K_{40}=9.98\times10^8)</td>
</tr>
<tr>
<td>Mass of foundation ( (N\cdotsec^2/m) )</td>
<td>(1.96\times10^6)</td>
</tr>
<tr>
<td>Inertia moment of foundation ( (kg\cdotm^2) )</td>
<td>(1.96\times10^8)</td>
</tr>
</tbody>
</table>

In this study, the optimal design of the TMD system is obtained for three soil types, namely the soft, medium and dense soil. The soil and foundation properties are presented in Table 2.

**Table 2: The properties of soil and foundation [31–32]**

<table>
<thead>
<tr>
<th>Type of soil</th>
<th>(K_r) ((N\cdots/m))</th>
<th>(K_s) ((N\cdots/m))</th>
<th>(C_r) ((N/m))</th>
<th>(C_s) ((N\cdots/m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft</td>
<td>(7.53\times10^{11})</td>
<td>(1.91\times10^9)</td>
<td>(2.26\times10^{10})</td>
<td>(2.19\times10^8)</td>
</tr>
<tr>
<td>Medium</td>
<td>(7.02\times10^{12})</td>
<td>(1.8\times10^{10})</td>
<td>(7.02\times10^{10})</td>
<td>(6.9\times10^8)</td>
</tr>
<tr>
<td>Dense</td>
<td>(1.91\times10^{13})</td>
<td>(5.75\times10^{10})</td>
<td>(1.15\times10^{11})</td>
<td>(1.32\times10^9)</td>
</tr>
</tbody>
</table>

The lower and upper bounds of the TMD parameters that are defined by Eq. (1) are shown in Table 3:

**Table 3: Upper and lower bands of the TMD parameters**

<table>
<thead>
<tr>
<th>Parameter of TMD</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_{TMD}) ((kg))</td>
<td>(M_{TMD}^{\text{min}} = 0.01 \times M_{\text{Structure}})</td>
<td>(M_{TMD}^{\text{max}} = 0.05 \times M_{\text{Structure}})</td>
</tr>
<tr>
<td>(C_{TMD}) ((N\cdots/m))</td>
<td>(2 \times 0.001 \times \frac{2\pi}{T_{TMD}^{\text{min}}} \times M_{TMD}^{\text{min}})</td>
<td>(2 \times 0.30 \times \frac{2\pi}{T_{TMD}^{\text{max}}} \times M_{TMD}^{\text{max}})</td>
</tr>
<tr>
<td>(K_{TMD}) ((N/m))</td>
<td>(\left(\frac{2\pi}{T_{TMD}^{\text{max}}}</td>
<td></td>
</tr>
</tbody>
</table><p>ight)^2 \times M_{TMD}^{\text{max}}) | (\left(\frac{2\pi}{T_{TMD}^{\text{min}}}ight)^2 \times M_{TMD}^{\text{min}}) |</p>

The optimal design of the TMD is found for the structure subjected to the Chi–Chi (CHY101) earthquake. This component of the recorded ground motion is shown in Fig. 2.
6. OPTIMIZATION RESULTS AND DISCUSSIONS

In this paper, the optimum design of TMD is obtained according to five different objective functions defined in section (2) for three soil types. For each objective function and soil type, twenty independent optimization is implemented. Finally, the results are shown in Tables (4–6).

Table 4: Optimal parameters of the TMD system for the structure located on the dense soil

| OF   | $K_{TMD}$ (N/m) | $C_{TMD}$ (N.s/m) | $M_{TMD}$ (kg) | $x_{Roof|SSI}$ (m) | $x_{Roof|SSI+TMD}$ (m) | $x_{TMD}$ (m) | Stroke ratio | Value of objective function |
|------|-----------------|------------------|---------------|-------------------|-----------------------|--------------|--------------|-----------------------------|
| OF 1 | 2.99×10^6       | 4.18×10^5        | 1.96×10^6     | 1.97              | 1.438                 | 5.376        | 1.9994       | 1.59329                     |
| OF 2 | 4.01×10^6       | 9.72×10^5        | 1.82×10^6     | 1.97              | 1.758                 | 4.590        | 1.4381       | 1.76117                     |
| OF 3 | 2.23×10^6       | 1.76×10^5        | 1.96×10^6     | 1.97              | 1.3059559             | 5.245        | 1.999984     | 1.38157                     |
| OF 4 | 2.23×10^6       | 1.76×10^5        | 1.96×10^6     | 1.97              | 1.3059554             | 5.245        | 1.999987     | 1.66352                     |
| OF 5 | 3.08×10^6       | 6.11×10^5        | 1.96×10^6     | 1.97              | 1.507                 | 4.879        | 1.71233      | 0.30386                     |

Table 5: Optimal parameters of the TMD system for the structure located on the medium soil

| OF   | $K_{TMD}$ (N/m) | $C_{TMD}$ (N.s/m) | $M_{TMD}$ (kg) | $x_{Roof|SSI}$ (m) | $x_{Roof|SSI+TMD}$ (m) | $x_{TMD}$ (m) | Stroke ratio | Value of objective function |
|------|-----------------|------------------|---------------|-------------------|-----------------------|--------------|--------------|-----------------------------|
| OF 1 | 2.58×10^6       | 3.88×10^5        | 1.96×10^6     | 2.054             | 1.4074                | 5.488        | 1.9874       | 1.54727                     |
| OF 2 | 3.71×10^6       | 1.07×10^5        | 1.96×10^6     | 2.054             | 1.7309                | 4.559        | 1.3771       | 1.75379                     |
| OF 3 | 2.06×10^6       | 1.51×10^5        | 1.96×10^6     | 2.054             | 1.330642              | 5.43760      | 1.99997     | 1.54196                     |
| OF 4 | 2.06×10^6       | 1.51×10^5        | 1.96×10^6     | 2.054             | 1.330641              | 5.43761      | 1.999977    | 1.64896                     |
| OF 5 | 3.06×10^6       | 5.29×10^5        | 1.96×10^6     | 2.054             | 1.51                  | 5.470        | 1.928487     | 0.33944                     |
Table 6: Optimal parameters of the TMD system for the structure located on the soft soil

| OF  | $K_{TMD}$\(^{(N/m)}\) | $C_{TMD}$\(^{(N.s/m)}\) | $M_{TMD}$\(^{(kg)}\) | $|\chi_{\text{SSI+TMD}}|_{(m)}$ | $|\chi_{\text{SSI+TMD}}|_{(m)}$ | Stroke ratio | Value of objective function |
|-----|---------------------|------------------------|-----------------|-----------------|-----------------|-------------|-----------------------|
| OF 1 | 1.59\times10^6      | 8.49\times10^5         | 1.96\times10^5  | 1.605           | 1.146938        | 4.3566      | 1.99999              | 1.48051          |
| OF 2 | 1.59\times10^6      | 8.49\times10^5         | 1.96\times10^5  | 1.605           | 1.146942        | 4.3564      | 1.99987              | 1.76063          |
| OF 3 | 1.51\times10^6      | 8.01\times10^5         | 1.96\times10^5  | 1.605           | 1.1455          | 4.3551      | 1.99994              | 2.79408          |
| OF 4 | 1.50\times10^6      | 7.95\times10^5         | 1.96\times10^5  | 1.605           | 1.1453          | 4.3550      | 1.99999              | 1.71960          |
| OF 5 | 1.59\times10^6      | 8.49\times10^5         | 1.96\times10^5  | 1.605           | 1.1469          | 4.3566      | 1.99997              | 0.61362          |

As can be seen from Tables (4–6), the optimal design of the TMD system is different for three soil types, while these parameters are almost the same for the soft soil. The objective function 4 leads to the minimum displacement of the roof story for the structure located on three soil types. Also, the objective function 2 leads to the minimum stroke ratio for the structure located on three soil types. The time history displacement of the roof story for the five objective functions and three soil types is depicted in Figs. (3-5).

Figure 3. Displacement time history of the roof story for the structure located on the dense soil

Figure 4. Displacement time history of the roof for the structure located on the medium soil
It can be observed from Figs. (3–5) that the SSI affect the optimal design of the TMD system. The time history acceleration of the roof story for the five objective functions and three soil types is also given in Figs. (6–8).
As can be seen from Figs. (6–8), the SSI effect can increase the acceleration of the roof story in comparison with that of the structure located on the fix base. Therefore, for the practical engineering problems, the SSI effects should be considered. The maximum absolute value for the displacement and acceleration of the roof story are reported in Table (7).

Figure 8. Acceleration time history of the roof story for the structure located on the soft soil

Table 7: The maximum absolute values of the displacement and acceleration for the roof story

<table>
<thead>
<tr>
<th>Soil type</th>
<th>OF 1</th>
<th>OF 2</th>
<th>OF 3</th>
<th>OF 4</th>
<th>OF 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dense</td>
<td>1.520057</td>
<td>1.856612</td>
<td>1.814780</td>
<td>1.381572</td>
<td>1.592386</td>
</tr>
<tr>
<td>Medium</td>
<td>4.844557</td>
<td>4.759222</td>
<td>5.198711</td>
<td>5.198714</td>
<td>4.822557</td>
</tr>
<tr>
<td>Soft</td>
<td>1.628412</td>
<td>1.997107</td>
<td>1.541958</td>
<td>1.541956</td>
<td>1.744889</td>
</tr>
</tbody>
</table>

As can be seen from Table (7), the soil type can change the seismic responses of the structure. In fact, for all considered soil type, the displacement of the roof is increased while the acceleration of the roof is decreased when the SSI state is compared with the fix base. For the dense, medium and soft soils, the minimum value of the roof displacement can be obtained based on OF 1, OF 4 and OF 4, respectively. While, the minimum value of the roof acceleration for the dense, medium and soft soils can be obtained based on OF 2, OF 2 and OF 1, respectively. Based on the above results, the type of soil and the optimization criterion (i.e. objective function) can effect the optimal design of TMD and the seismic responses of the structure.
The transfer function for the relative displacement and the absolute acceleration of the roof story is computed by the equations:

\[
\begin{align*}
TF_{ACC. Roof} & = \frac{TF_{with\ TMD}}{TF_{without\ TMD}} \\
TF_{Disp. Roof} & = \frac{TF_{with\ TMD}}{TF_{without\ TMD}}
\end{align*}
\] (33) (34)

For the dense soil, the transfer function of the displacement and absolute acceleration for the roof storey of the controlled structure obtained using the five objective functions is depicted in Figs. (9) and (10).

Figure 9. The transfer function of the absolute acceleration for the structure located on the dense soil

Figure 10. The transfer function of the relative displacement for the structure located on the dense soil
It can be concluded from Figs. (9) and (10) that for the dense soil, the OF 5 has the minimum value, while the OF (2) and (4) have the maximum values for the transfer function of the absolute acceleration and relative displacement, respectively. These transfer functions for the controlled structure located on the medium soil are depicted in Figs. (11) and (12).

![Graph of absolute acceleration transfer function](image1)

**Figure 11.** The transfer function of the absolute acceleration for the structure located on the medium soil.

![Graph of relative displacement transfer function](image2)

**Figure 12.** The transfer function of the relative displacement for the structure located on the medium soil.

As can be observed from Figs. (11) and (12), the minimum value is obtained based on the OF 5. These values for the controlled structure located on the soft soil are also given in Figs. (13) and (14). Based on these figures, it can be concluded that there are no difference between the response of the different objective functions for the relative displacement and absolute acceleration.
7. CONCLUSIONS

In this study, the five different optimization criteria in terms of the reduction of structural responses (including the displacement and acceleration) were compared in order to find the optimal design of the TMD system for a 40–storey shear building with the SSI effect and the limitation of the scaled stroke of TMD. The WOA was used to optimize the parameters of a TMD system (including the mass, damping and stiffness) subjected to earthquake load.

The results reveal the SSI effect can increase the seismic responses of the structure in comparison with those of the structure located on a fix base. Therefore, the SSI effects should be considered in the optimal design of the TMD system. Furthermore, it can be found that the maximum of the roof displacement as the objective function can be adopted in the
optimal design of the TMD system for the controlled structure located on the soft and medium soil. Although, minizing the acceleration of the controlled structure is considered, the best objective function can differ from minizing the roof displacement of the controlled structure. Therefore, the soil type and the choice of the objective function have the important roles in the optimal design of the TMD system.

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REFERENCES