THE PERIODIC GREEN VEHICLE ROUTING PROBLEM WITH CONSIDERING OF TIME-DEPENDENT URBAN TRAFFIC AND TIME WINDOWS

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ABSTRACT

The travel times among demand points are strongly influenced by traffic in a supply chain. Due to this fact, the service times for customers are variable. For this reason, service time is often changes over a time interval in a real environment. In this paper, a time-dependent periodic green vehicle routing problem (VRP) considering the time windows for serving the customers and multiple trip is developed with this assumption that urban traffic would disrupt timely services. The objective function of proposed problem is to minimize the total amount of carbon dioxide emissions produced by the vehicle, earliness and lateness penalties costs and costs of used vehicles. At first, a novel linear integer mathematical model is formulated and then the model is validated via solving some test problems by CPLEX solver. Finally, the sensitivity analysis is carried out to study the role of two critical parameters in the optimal solution.

Keywords: periodic green vehicle routing problem; time-dependent urban traffic; multiple trip; carbon dioxide emission; time window.

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1. INTRODUCTION

Increasing the energy prices has profound effects on the public and industrial transportation systems so that to decrease the costs in order to survive in this competitive environment has the highest priority in most distribution centers. On the other hand, traffic status become more complicated in metropolises with increasing the number of automobiles, road construction and so on. Accordingly, to establish the optimal routes for serving on time and minimize total cost for a supply chain is a key and complex problem. Notably, the travel time among demand points is dependent on not only distance but traffic status.

One of paper objectives is to develop a vehicle routing problem (VRP) to minimize both costs and travel times.

On the other hand, it is obvious that carbon dioxide (CO$_2$) has had the main role for global warming in recent decades. In 1958, it was founded that carbon density is increasing in earth's atmosphere widely [1]. According to the International Energy Agency (IEA) reports, the transportation sector has the second main role for CO$_2$ emission after the generation of electricity and thermal energy that this amount has been 22% of total CO$_2$ emission in 2010 and almost One-third of it is associated to transportation sector [2]. Traffic congestion and density leading decreasing of movement velocity and also fluctuations in velocity as acceleration can play a substantial role in CO$_2$ emission and generation [3, 4]. Therefore, reducing the consumption of fossil fuels and CO$_2$ emission produced in roads can be controlled by optimizing the transportation operations. The goal of article is to present a new mathematical model and optimizing model for reducing the CO$_2$ emission with using the VRP problem considering the traffic congestion.

VRP problem has a historical and theoretical context in traveling salesman problem (TSP) introduced by Dantzig and Ramser [5]. VRP can determine the optimal delivery routes or collection from one/multiple depot toward the costumers together with a set of constraints. The objective is to find routes minimizing the total travel time (distance, cost, number of vehicle). Hajishafee et al. presented a capacitated vehicle routing problem (CVRP) with vehicles hire or purchase decision [6]. They formulated the problem as a mixed integer programming (MIP) model in which the sum of net present value (NPV) of procurement and traveling costs is minimized.

The different practical versions of VRP have been proposed in literature such as VRP with multi-depot, together with time windows, time-dependent VRP, Split delivery VRP and etc. In recent decade, green VRP [7, 8] which was introduced with the aim of balancing the operating and environmental costs attracts many researchers’ attention. Erikson et al. investigated the effect of traffic on increasing fuel consumption and developed a mathematical model to estimate the potential reduction in fuel consumption by optimized routing [9]. Kara et al. presented a cost function consists of energy consumption for VRP and they introduced it as Energy Minimization for VRP (EMVRP) minimizing the total energy consumption during routs (instead of total travelled distance) in order to reduce the fuel consumption and CO$_2$ emission [10]. Coaw presented a mathematical model to calculate total consumed fuel for a time-dependent VRP so that he considered property of effective load weight in consumption [11]. Goor et al. studied cumulative VRP that the vehicles have
pre-determined capacities and number of evacuation camps with objective of minimizing the fuel consumption. They solved this model by an approximation algorithm [12]. Franschetti et al. developed a time-dependent VRP considering the traffic density [4]. In their method, the planning horizon has been divided to 2 periods 1) the initial period with less traffic and fixed velocity of vehicles and 2) the next period with velocity changing and linear dependence travelling time on start time of travel.

The varying conditions of traffic congestion means that road networks can be attributed with time-dependent characteristics, and the vehicle routing problem (VRP) that deals with these characteristics is called time-dependent VRP (TDVRP) [13, 14]. Unlike distance-based VRP, TDVRP considers the time of departure from each node as a decision variable. In TDVRP, the total travel time depends on the time of departure from nodes and objective functions usually seek to minimize the total travel time, fuel consumption or CO2 emissions.

VRP with time windows (VRPTW) is a variant of VRP in which each customer must be served within a specified time window. Practical examples of VRPTW include distribution of cash among bank branches, industrial waste collection, distribution of fuel among gas stations, school bus service, etc. VRPTW have also been used in many studies focusing on subjects such as food supply chain, garbage collection, etc. [15, 16].

There are several various works regarding the network design for transportation networks such as work of Afrazandeh-Zargari et al. [17]. They have presented an Ant Colony System (ACS) based algorithm to solve the Time Dependent Network Design Problem (T-NDP). They used it to solve different networks with different size. Since the subject of the current paper is different from network design, more investigating in this subject is disregarded.

Most vehicle routing problems consider an average speed for vehicles traversing between demand points but ignore the dynamic environmental factors such as the traffic condition.

This paper presents a green time-dependent VRP with time windows, whose objective is to minimize total CO2 emissions, total travel time, and total delay. In this study, road network are defined based on several traffic densities, and planning horizon consists of multiple periods.

Unlike most green time-dependent VRPs which only consider the departure times as decision variable, the problem proposed in this paper considers: 1) several planning period for each arc defined in the network, 2) the total distance of the arcs traveled in each planning period, and 3) optimal departure time and arrival time for each arc in each period.

Other innovation of this paper is the incorporation of customer time windows, heterogeneous fleet of vehicles with different capacities, and multiple trips.

In the remainder of this paper: Section 2 defines the problem assumptions and presents the mathematical formulations, Section 3 discusses the computation results and sensitivity analysis, and finally Section 4 presents the conclusion and some suggestions for future work.

2. PROBLEM DEFINITION

Consider a business in which a heterogeneous fleet of vehicles must serve n customers
scattered throughout the supply chain network by starting from a depot and returning to the same depot after the end of the operation. By definition, $V$ is the set of available heterogeneous vehicles, $W_v$ is the capacity of vehicle $v$, and $TM_v$ is the maximum operation time available for each vehicle, which determines the allowed number of trips in its tour. Problem is defined as a complete graph $G (N, A)$ where $N=\{0,1,2,...,n\}$ is the set of all nodes and $N' \subseteq N$ is the set of customers. Arc $(i, j)$ represents a path from node $i$ to node $j$, and length of arc $(i, j)$ is denoted by $C_{ij}$. Customer $i$ has a demand $D_i$, a primary time window $[e_i, l_i]$ and a secondary time window $[e_e, l_l]$, which also includes the duration of service; violation of primary time window is not allowed but secondary time window can be violated by applying an earliness and tardiness penalty. The time horizon of problem is divided into $m$ periods, during each of which traffic condition and vehicle speed are constant (different periods may have different traffic conditions and vehicle speeds). The set of time period is $P$, where $p \in P$ and start and end of each period are denoted by $b_p$ and $g_p$ respectively.

The speed of vehicles traversing the arc $(i, j)$ in the period $p$ is $S_{ijp}$, which depends on traffic information. The rate of CO2 emission (Lb / mile) by vehicle $v$ traversing the arc $(i, j)$ in the period $p$ is $h_{ijpv}$ and is calculated via emission models available the literature. Note that all variables associated with roads and vehicles including speed, road gradient, traffic density, and vehicle weight, are involved in the calculation of $h_{ijpv}$.

The proposed problem consists of four basic decisions for each vehicle: 1) which vehicle should be used in each period 2) What would be the optimal route for the used vehicle to service all the customers. 3) What would be the departure time and arrival time at each node in the constructed tour of vehicles. 4) What distance should be traveled to cover the optimal tour.

The objective function of the problem seeks to minimize the total volume of CO2 emissions, the total time of service, and incurred earliness and tardiness penalties. The proposed problem is more generalized than the traditional TDVRP in the literature, as it can consider the vehicle idle time at any point on the tour to avoid traffic congestion.

Fig. 1 shows a schematic description of the problem. In this figure, we see one depot and ten customers in the network. Two type of vehicle are available in depot with each vehicle being allowed to make three trips at most.

The proposed problem is based on a set of assumptions and conditions which are described in the following subsection.

### 2.1 Definitions and assumptions of the proposed model

In this section we present the assumption of the problem.

- Fleet of vehicles is heterogeneous and vehicles have different capacities.
- Each node (customer) is served only once by one vehicle and split delivery is not allowed.
- Each customer a time window composed of two intervals: primary and secondary. Secondary time window can be violated by applying an earliness and tardiness penalty but violation of primary time window is not allowed.
- Graph consists of a single depot and a set of customers.
- Traffic condition of every route is known. This condition determines the speed of vehicles.
- Each vehicle can have multiple trips in its tour.
- All vehicles start their tour from the depot, and after the end of their trip(s), return to the depot.
- Rate of CO2 emission—per mile of trip—depends on road and vehicle conditions and is known.
- Each vehicle has a maximum operation time.
- Distances between points on the network are Euclidean.
- Time and cost of traversing a certain route are identical for all vehicles.

Figure 1. A scheme for the proposed problem

2.2 Sets and parameters

The notations of parameters and sets used in the mathematical model are presented in this section.

\[ i, j \] : Notation of the set of all nodes \((i, j \in N)\)
Notation of the set of vehicles \((v \in V)\)

Notation of the set of planning periods \((p \in P)\)

Notation of the trips \((r \in R)\)

The number of planning periods

Time of loading at the depot node

Time of unloading at the demand node

A factor for conversion of CO2 emission to cost ($ / Lb)

Time of travel from node \(i\) to node \(j\) by vehicle \(v\) in planning period \(p\); equals to

\[
\frac{C_{ij}}{S_{ijp}}
\]

Rate of CO2 emission (Lb/mile) for vehicle \(v\) traversing the arc \((i,j)\) in planning period \(p\)

Speed of vehicle \(v\) traversing the arc \((i,j)\) in planning period \(p\) (mile/hour)

Distance of travel from node \(i\) to node \(j\)

Demand of customer \(i\); \(D_0 = 0\)

Earliness penalty

Tardiness penalty

Lower bound of primary time window of customer \(j\)

Upper bound of primary time window of customer \(j\)

Lower bound of secondary time window of customer \(j\)

Upper bound of secondary time window of customer \(j\)

Operation cost of vehicle \(v\) in planning period \(p\)

Maximum operation time of vehicle \(v\)

Capacity of vehicle \(v\)

A very large number

Total loading time of vehicle \(v\) in trip \(r\)

Total unloading (service) time at demand node \(j\)

2.3 Decision variables

\(x_{ij}\) A binary variable; \(x_{ij} = 1\) if arc \((i,j)\) is traversed; \(x_{ij} = 0\) otherwise

\(y_{ijv}\) A binary variable; \(y_{ijv} = 1\) if arc \((i,j)\) is traversed by vehicle \(v\) in trip \(r\); otherwise \(y_{ijv} = 0\)

\(xx_{ijvpr}\) A binary variable; \(xx_{ijvpr} = 1\) if arc \((i,j)\) is traversed by vehicle \(v\) in trip \(r\) and planning period \(p\); otherwise \(xx_{ijvpr} = 0\)

\(U_{vp}\) A binary variable; \(U_{vp} = 1\) if vehicle \(v\) is used in planning period \(p\); otherwise \(U_{vp} = 0\)

\(z_{ijvpr}\) A continuous variable; represents the distance traveled by vehicle \(v\) moving from node \(i\) to node \(j\) in trip \(r\) and planning period \(p\)
\[ \pi_{ijvp} \] A continuous variable; represents the travel time of vehicle \( v \) moving from node \( i \) to node \( j \) in trip \( r \) and planning period \( p \)

\[ dp_i \] A continuous variable; represents the time of departure from node \( i \)

\[ ar_i \] A continuous variable; represents the time of arrival at node \( i \)

\( YE_i \) The amount of earliness in serving customer \( i \)

\( YL_i \) The amount of tardiness in serving customer \( i \)

2.5 Mathematical model

\[
\text{Minimize } Z_1 = \alpha \left( \sum_{i \in N} \sum_{j \in N} \sum_{p \in P} \sum_{v \in V} z_{ijvp} h_{ijvp} \right) + \sum_{v \in V} \sum_{p \in P} CK_{vp} U_{vp} + \sum_{i \in N} (PE \cdot YE_i) 
\]

Subject to:

\[ \sum_{j \in N} x_{ij} = 1 \quad \forall i \in N' \] (2)

\[ \sum_{i \in N} x_{ij} = 1 \quad \forall j \in N' \] (3)

\[ x_{ij} = \sum_{v \in V} \sum_{r \in R} y_{ijvr} \quad \forall (i, j) \in A \] (4)

\[ y_{ijvr} \geq \alpha x_{ijvp} \quad \forall (i, j) \in A, v \in V, p \in P, r \in R \] (5)

\[ y_{ijvr} \leq \sum_{p \in P} \sum_{r \in R} xx_{ijvp} \quad \forall (i, j) \in A, v \in V \] (6)

\[ \sum_{i \in N \setminus j} y_{ijvr} = \sum_{i \in N \setminus j} y_{ijvr} \quad \forall v \in V, r \in R, j \in N \] (7)

\[ \sum_{j \in N} y_{0jvr} \leq 1 \quad \forall v \in V, r \in R \] (8)

\[ z_{ijvp} \leq C_{ij} \sum_{r \in R} xx_{ijvp} \quad \forall (i, j) \in A, v \in V, p \in P \] (9)

\[ \sum_{p \in P} \sum_{v \in V} z_{ijvp} = x_{ij} \cdot C_{ij} \quad \forall (i, j) \in A \] (10)

\[ \pi_{ijvp} = \frac{z_{ijvp}}{S_{ijp}} \quad \forall (i, j) \in A, v \in V, p \in P \] (11)
\[ \sum_{(i,j) \in A} \pi_{ijvp} \leq g_p - b_p \quad \forall v \in V, p \in P \] (12)

\[ dp_i \leq g_p - \pi_{ijvp} + g_m \left( 1 - \sum_{r \in R} xx_{ijpvr} \right) \quad \forall (i, j) \in A, v \in V, p \in P \] (13)

\[ ar_j \geq b_p + \pi_{ijvp} - g_m \left( 1 - \sum_{r \in R} xx_{ijpvr} \right) \quad \forall (i, j) \in A, v \in V, p \in P \] (14)

\[ ar_j \geq dp_i + \sum_{v \in V} \sum_{p \in P} \pi_{ijvp} - g_m \left( 1 - x_{ij} \right) \quad \forall (i, j) \in A \] (15)

\[ ar_i + UT_i \leq dp_i \quad \forall i \in N' \] (16)

\[ ar_0 \leq g_m \] (17)

\[ \sum_{j \in N} \sum_{i \in N} D_j y_{ijv} \leq W_v \quad \forall r \in R, \forall v \in V \] (18)

\[ \sum_{r=1}^{R} LT_v^r + \sum_{j \in N} UT_j + \sum_{i \in N} \sum_{j \in N} \sum_{p \in P} \sum_{r \in R} t_{ijp} xx_{ijpvr} \leq TMv \quad \forall v \in V \] (19)

\[ LT_v^r = ul \cdot W_v \cdot \sum_{(i,j) \in A} y_{ijv} \quad \forall v \in V, \forall r \in R \] (20)

\[ UT_j = uu \sum_{v \in V} \sum_{i \in N} \sum_{r \in R} D_j y_{ijv} \quad \forall j \in N' \] (21)

\[ \sum_{i \in N} \sum_{j \in N} \sum_{r \in R} xx_{ijpvr} \leq MM \cdot U_{vp} \quad \forall v \in V, p \in P \] (22)

\[ \sum_{j \in N} xx_{0jpv} \geq \sum_{j \in N} xx_{0jp v r+1} \quad \forall v \in V, p \in P, \forall r = 1, 2, \ldots, R - 1 \] (23)

\[ e_i \leq ar_i \leq l_i \quad \forall i \in N' \] (24)

\[ YE_v \geq ee_i - ar_i \quad \forall i \in NC \] (25)

\[ YL_i \geq ar_i - l_i \quad \forall i \in NC \] (26)

\[ x_{ijk r}^m \in \{0,1\}, u_{ik} \in \{0,1\}, \forall i = 1, 2, \ldots, V, \forall j = 1, 2, \ldots, V, \forall k = 1, 2, \ldots, K, \forall r = 1, 2, \ldots, R, \forall m = 1, 2, \ldots, M \] (27)
Objective function seeks to minimize the total volume of CO2 emissions (converted to cost by the use of factor $a$), the incurred earliness and tardiness penalties, and the vehicle operation cost over the planning period. Constraints (2) and (3) ensure that each customer is served only once. Constraint (4) ensures that each arc is traversed by only one vehicle. Constraints (5) and (6) state that $y_{ijvr}$ should be equal with $xx_{ijpv}$. Constraint (7) ensures a balance of flow for vehicles, meaning that in each trip, the number of vehicles entering and exiting any given node should be equal. Constraint (8) indicates that in each trip each vehicle leaves the depot node at most once. Constraint (9) ensures that if $xx_{ijpv} = 0$ then $z_{ijvp} = 0$, and that if $xx_{ijpv} = 1$ then $z_{ijvp}$ is limited to $C_{ij}$. Constraint (10) indicates that when arc $(i, j)$ is selected, its total distance must be traveled. Constraint (11) determines the travel times associated with variable $z_{ijvp}$. Constraint (12) indicates that the total travel time of each vehicle in each period must be smaller than the length of that period. Constraint (13) states that if arc $(i, j)$ is traversed in period $p$, time of departure from node $i$ should be smaller than the end time of period $p$ minus the time required to travel the arc $(i, j)$ in that period. Similarly, constraint (14) states that time of arrival at node $j$ should be greater than or equal the start time of period $p$ plus the time required to travel the arc $(i, j)$ in that period. Constraints (12) and (13) activate when $xx_{ijpv} = 1$. When $i = 0$ constraints (12) and (13) remain satisfied because value of $g_m$ is positive and sufficiently large to ensure that these inequalities remain satisfied. Constraints (15) determines the soonest time of arrival at node $j$. Constraint (16) states that vehicle departure time equals its arrival time plus the time required to serve customer $i$. Constraints (15) and (16) also lead to elimination of sub-tours. Constraint (17) states that the time of return to the depot should not exceed (be later than) the end time of last period. Constraints (18) and (19) represent the capacity limit (in each trip) and time limit of vehicles. Relationships (20) and (21) determine the loading/unloading time of vehicles. Constraint (22) states that vehicle $v$ can be used only when its cost is paid. Constraint (23) represents the necessity of compliance with the sequence of trips $r$ to $r + 1$. Constraint (24) ensures that primary time windows of customers remain unviolated. Constraints (25) and (26) determine the duration of earliness and tardiness. Constraint (27) specifies the type of variables.

3. NUMERICAL RESULTS

To validate the proposed model, first 10 instances are generated randomly, and they are then solved with CPLEX solver provided in GAMS software. In the second subsection, sensitivity of important parameters and their impact on the total cost are evaluated through a sensitivity analysis performed on demand ($D_i$) and maximum operation time of vehicles ($T_{MV}$).

3.1 Generation of random Instances

In this subsection, 10 random instances are generated. Data details of these problems are shown in Table 1. Other information and assumptions of these random problems are described below.
Each working hour of each day are divided into five periods \((m=5)\): the first period is from 6 am to 8 am, the second period is from 8 am to 10 am, the third period is from 10 am to 12 am, the fourth period is from 12 am to 2 pm, and the last period is from 2 pm to 5 pm. Each of these periods has its own vehicle speed and CO2 emission specifications: for the first period \(S_{ijp} \sim U[35,50]\) and \(h_{ijpv} \sim U[1,2]\), for the second period \(S_{ijp} \sim U[30,45]\) and \(h_{ijpv} \sim U[1.5,2.5]\), for the third period \(S_{ijp} \sim U[25,40]\) and \(h_{ijpv} \sim U[3,3.5]\), for the fourth period \(S_{ijp} \sim U[15,25]\) and \(h_{ijpv} \sim U[4,4.5]\) and for the fifth period \(S_{ijp} \sim U[25,45]\) and \(h_{ijpv} \sim U[2,3.5]\). It should be mentioned that vehicles have a constant rate of emission.

In these problems, fleet is composed of two types of vehicles: vehicle type-I with capacity of 1 ton, and vehicles type-II with capacity of 2 tons. The loading time of vehicles type-I and type-II is, respectively, 0.75 minutes and 1.25 minutes and their unloading time is, respectively, 1.75 minutes and 2.25 minutes. The cost of operation of vehicles type-I and type-II in each period is, respectively 500 units and 600 units (of currency). The value of parameter \(a\) is considered to be 10 units. These problems have a uniformly distributed demand \(D_i \sim U[50,100]\) and the maximum operation time of vehicles \(TV\) is assumed to be the fixed value of 8 hours. The distance of each arc is \(C_{ij} \sim U[0,3,3]\), which can be used to calculate \(t_{ijpv}\). Earliness penalty \(PE\) is equal to 100 units and tardiness penalty \(PL\) is 200 units. The generated problems are solved with GAMS software and the results, including the value of objective function and the solution time, are presented in Table 2.

Notice, a 3600 seconds run time limit is considered as stopping criteria and the best found solution is reported.

In Table 1, the first column is the problem number, the second column is the number of nodes in the problem, the third column shows the number of available type-I vehicles, the fourth column shows the number of available type-II vehicles.

<table>
<thead>
<tr>
<th>Problem</th>
<th>n</th>
<th>V1</th>
<th>V2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>P2</td>
<td>7</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>P3</td>
<td>8</td>
<td>2</td>
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</tr>
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<td>P4</td>
<td>10</td>
<td>3</td>
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<td>P5</td>
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<td>4</td>
<td>3</td>
</tr>
<tr>
<td>P7</td>
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<td>4</td>
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</tr>
<tr>
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<td>25</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>P9</td>
<td>30</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>P10</td>
<td>35</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

In Table 2, the first column is the problem number, second column presents the value of objective function obtained in each problem, and the last column shows the solution time, i.e. the execution time of GAMS software.
Table 2: Obtained results by solving instances

<table>
<thead>
<tr>
<th>Problem</th>
<th>Value of objective function</th>
<th>Solution time (seconds)</th>
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</thead>
<tbody>
<tr>
<td>P1</td>
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<tr>
<td>P2</td>
<td>2120</td>
<td>6.2</td>
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<td>P3</td>
<td>2592</td>
<td>6.8</td>
</tr>
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<td>P4</td>
<td>3306</td>
<td>24.1</td>
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<tr>
<td>P5</td>
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<td>92.5</td>
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<td>P6</td>
<td>4851</td>
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<td>P7</td>
<td>5102</td>
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<td>P8</td>
<td>6002</td>
<td>1916.1</td>
</tr>
<tr>
<td>P9</td>
<td>6653</td>
<td>2567.3</td>
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<tr>
<td>P10</td>
<td>8230</td>
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<tr>
<td>Average</td>
<td>4439.5</td>
<td>922.22</td>
</tr>
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3.1 Sensitivity analysis

To assess the impact of important parameters on the value of objective function, two parameters of demand and maximum operation time of vehicles, which are the most important problem parameters, are subjected to a sensitivity analysis. The variation limit of demand is assumed to be $D_i = [0.8D_i, D_i, 1.2D_i]$ and the maximum operation time is assumed to be 6 hours, 8 hours, 9 hours and 10 hours. Tables 2 and 3 show a summary of the results of sensitivity analysis.

Table 2: Sensitivity analysis of Demand

<table>
<thead>
<tr>
<th>Problem</th>
<th>Demand $D_i$</th>
<th>$0.8D_i$</th>
<th>$D_i$</th>
<th>$1.2D_i$</th>
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<td>P10</td>
<td>7581</td>
<td>8230</td>
<td>10626</td>
<td></td>
</tr>
</tbody>
</table>

Examining Table 2 and Fig. 2 shows that a 20% decrease in demand makes no significant change in the value of objective function, however, changes made in this parameter because of 20% increase in demand are significant and increase with the increase of problem size. This result highlights the importance of optimal policy-making under varying demand condition.
As Table 3 and Fig. 3 show, the values of objective function for $TM_v$ of 6 hours are significantly greater than the values obtained for $TM_v$ of 8 hours, and this difference increases with the increase of problem size. The difference between the results of $TM_v=9$ hours and results of $TM_v=10$ hours is somewhat negligible; but the difference between $TM_v=8$ hours and $TM_v=9$ hours is slightly more significant. In fact, decision should be made with due consideration given to the best maximum operation time in terms of objective function value.

Table 3: Sensitivity analysis of maximum operation time of vehicles

<table>
<thead>
<tr>
<th>Problem</th>
<th>Maximum operation time of vehicles $TM_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6 hours</td>
</tr>
<tr>
<td>P1</td>
<td>1720</td>
</tr>
<tr>
<td>P2</td>
<td>2876</td>
</tr>
<tr>
<td>P3</td>
<td>3376</td>
</tr>
<tr>
<td>P4</td>
<td>4206</td>
</tr>
<tr>
<td>P5</td>
<td>5461</td>
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<td>P6</td>
<td>6148</td>
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<tr>
<td>P7</td>
<td>6576</td>
</tr>
<tr>
<td>P8</td>
<td>7814</td>
</tr>
<tr>
<td>P9</td>
<td>8488</td>
</tr>
<tr>
<td>P10</td>
<td>11223</td>
</tr>
</tbody>
</table>
4. CONCLUSION

The aim of this study was to present an efficient model for optimization the logistics and supply chain systems with incorporation of transport and environmental conditions. To achieve this goal, we developed a mixed integer linear mathematical programming model for the green vehicle routing problem with time windows and time-dependent urban traffic. This model determines the optimal route, the travel schedules of vehicles, and the number of required vehicles with respect to time windows defined for customers and with the objective of minimizing the total operation cost and the total volume of CO2 emission. Then, CPLEX solver was applied to solve a number of randomly generated instances of the problem and then two important model parameters were subjected to sensitivity analysis where the results of which were presented to facilitates management in decisions making process. Future studies may be devoted to incorporate the uncertain nature of demand and traffic conditions into modeling and to use efficient meta-heuristic algorithms for larger variants of this problem.

REFERENCES
