STRUCTURAL OPTIMIZATION USING BIG BANG-BIG CRUNCH ALGORITHM: A REVIEW

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ABSTRACT

The big bang-big crunch (BB-BC) algorithm is a popular metaheuristic optimization technique proposed based on one of the theories for the evolution of the universe. The algorithm utilizes a two-phase search mechanism: big-bang phase and big-crunch phase. In the big-bang phase the concept of energy dissipation is considered to produce disorder and randomness in the candidate population while in the big-crunch phase the randomly created solutions are shrunk into a single point in the design space. In recent years, numerous studies have been conducted on application of the BB-BC algorithm in solving structural design optimization instances. The objective of this review study is to identify and summarize the latest promising applications of the BB-BC algorithm in optimal structural design. Different variants of the algorithm as well as attempts to reduce the total computational effort of the technique in structural optimization problems are covered and discussed. Furthermore, an empirical comparison is performed between the runtimes of three different variants of the algorithm. It is worth mentioning that the scope of this review is limited to the main applications of the BB-BC algorithm and does not cover the entire literature.

KEY WORDS: structural optimization; metaheuristic techniques; big bang-big crunch algorithm; global optimization; stochastic search; optimal design

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1. INTRODUCTION

Over the years, the drawbacks of traditional structural optimization methods namely mathematical programming [1] and optimality criteria [2, 3] techniques (such as their gradient based formulations and inefficiency in handling discrete design variables) have resulted in an increasing tendency towards stochastic search techniques or the so-called metaheuristics. Generally, metaheuristic techniques, such as genetic algorithms (GAs) [4],

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simulated annealing (SA) [5], particle swarm optimization (PSO) [6], ant colony optimization (ACO) [7, 8], harmony search method (HS) [9], etc. employ non-deterministic search strategies by borrowing their working principles from natural phenomena [10]. Numerous applications of metaheuristics accumulated up to date in structural design optimization can be attributed to their promising solutions, independency on gradient information, and capability of handling both continuous and discrete design variables. The state-of-the-art reviews of these algorithms as well as their applications in structural design optimization problems are outlined in Refs. [11, 12].

Generally, both the trajectory and population based metaheuristic optimization algorithms aim to find the global optimum in the solution space through random moves. The strategy by which an algorithm proposes the next move in the solution space can be considered as its main feature. Here, the employed strategies and mechanisms for proposing more reliable moves in the solution space become crucial. This fact motivates the developers of optimization algorithms to find more efficient methodologies for proposing robust optimization algorithms. However, sometimes this attempt yields complicated approaches which are difficult to understand and implement. In fact, in engineering optimization applications there is a great demand for efficient optimization algorithms having simple algorithmic structure [13].

A good example of algorithmic simplicity, yet efficiency, is the BB-BC algorithm proposed by Erol and Eksin [14] in 2006. The BB-BC algorithm is a metaheuristic optimization technique which is based on one of the theories for the evolution of the universe. The algorithm utilizes a two-phase search mechanism: big-bang phase and big-crunch phase. In the big-bang phase the concept of energy dissipation is considered to produce disorder and randomness in the candidate population while in the big-crunch phase the randomly created solutions are shrunk into a single point in the design space. It is worth mentioning that this mechanism enables the algorithm to be more explorative at the initiation of the optimization process and gradually limits its search towards more reliable regions of the solution space in the last iterations. The above mentioned two-phase search scheme is repeated until a predetermined termination criterion is met.

The BB-BC optimization algorithm has gained significant popularity in the recent years basically because of its simplicity and acceptable performance in solving engineering optimization problems. This review study is an attempt to identify and summarize the latest promising applications of the BB-BC algorithm in optimal structural design. Different variants of the algorithm as well as strategies to reduce the total computational effort of the technique in structural optimization problems are covered and discussed. It is worth mentioning that the scope of this review study is limited to the main applications of the BB-BC algorithm and does not cover the entire literature. The remaining sections of the paper are organized as follows. The second section briefly describes the algorithmic structure of the BB-BC algorithm. A survey of the recent applications of BB-BC algorithm in structural design optimization is presented in the third section. In the fourth section an empirical comparison is performed between the runtime of three different variants of the BB-BC algorithm. The last section provides the concluding remarks.
2. DESCRIPTION OF THE BB-BC ALGORITHM

This section briefly describes the main steps for implementation of the BB-BC algorithm. As mentioned before, the big bang-big crunch method is a population based optimization algorithm emerged from the big bang and big crunch theories of the universe evolution. As its name implies, the algorithm is based on the continuous application of two successive stages, i.e. big bang and big crunch phases. During big bang phase, new solution candidates, which are the parameters that affect the fitness function, are randomly generated around a “center of mass”, which is later computed in the big crunch phase based on their fitness values. The steps in the implementation of the BB-BC algorithm are outlined as follows [15]:

i. Form the initial population by spreading randomly solution candidates over all search space (first big bang) in a uniform manner. This step has to be applied once.

ii. Calculate the fitness value of every individual point and assign this value as its mass (if a minimization is to be carried out, form the “mass value” either by inversing the fitness/cost value or by subtracting it from a constant number chosen bigger than the maximum possible value).

iii. Calculate the “center of mass” by taking the weighted average using the coordinates and the mass values of every single individual (big crunch phase) or choose the fittest individual among all as their center of mass.

iv. Generate new solution candidates by using normal distribution (big bang phase).

v. Keep the fittest individual found so far in a separate place or as a member of the population (elitism) and go to step ii until a stopping criterion is accomplished.

There are different variants of the BB-BC available in the literature, for instance in Ref. [15] at each iteration of the BB-BC algorithm equation (1) is used to generate the new candidates around the center of mass which is taken as the fittest individual of the population.

\[ x_i^{new} = x_i^c + \alpha r_i \frac{(x_i^{max} - x_i^{min})}{k} \]  

where is the value of i-th design variable in the fittest individual, and are the lower and upper bounds of the i-th variable, respectively, is a randomly generated number according to a standard normal distribution, \( k \) is the iteration number, and \( \alpha \) is a constant.

3. STRUCTURAL OPTIMIZATION APPLICATIONS

A brief review on the applications of the BB-BC algorithm in structural optimization is presented in this section. As mentioned before, the BB-BC optimization technique is first proposed in 2006 by Erol and Eksin [14]. One of the very first applications of the BB-BC algorithm in structural optimization was carried out by Camp [16] in 2007. Camp [16] used this optimization technique for weight minimization of several truss structures subjected to stress and displacement constraints. The performance of the original algorithm was improved by introducing a weighting parameter that controlled the influence of the center of
mass and the current global best solution on new candidate solutions as well as application of a multiphase search strategy. Planar and three-dimensional truss structures were optimized considering the discrete and continuous design variables. It was concluded from the results that the BB–BC algorithm seems to show significant improvements in the consistency and computational efficiency compared to GA, PSO, and ACO techniques. It was also shown that the BB-BC method can handle continuous and discrete optimization problems efficiently.

In 2009, Kaveh and Talatahari [17] utilized the BB-BC algorithm for size optimization of space truss structures. An improved BB-BC algorithm called the hybrid BB–BC algorithm was proposed. The proposed HBB-BC method considers the combination of the center of mass, the best position of each candidate and the best visited position of all candidates as an average point in the beginning of each big-bang phase. This is similar to the PSO method and increases the exploration capacity of the algorithm. In addition to this a Sub-Optimization Mechanism (SOM), based on the principles of the finite element method, was employed to further improve the performance of the method. Based on the results obtained from optimization of several space trusses using HBB-BC algorithm, it was concluded that the new method has better performance compared to GA, ACO, PSO and even better than HS, in some cases. It was demonstrated that, in contrast to the other metaheuristic techniques which may encounter premature convergence in large problems, HBB-BC performs well in large size structures. However, the need for an additional local search scheme for the proposed HBB-BC algorithm was also emphasized.

Kaveh and Talatahari [18] conducted a research in 2010 on the optimum design of Schwedler and ribbed domes via HBB-BC algorithm. Optimal joint coordinates and member connectivity is found for each problem considering the geometrical nonlinearity effects in the analysis. Three load cases (vertical, horizontal, and combination of these two) were considered in the design phase. The HBB-BC method with discrete variables was utilized and satisfactory results were found. A comparison between the Schwedler and ribbed domes was made for different load cases. It was also reported that for Schwedler domes selecting a ratio of the height to the diameter from the range of [0.2, 0.4] can improve the performance of the dome. In addition the results revealed that the normalized required material for Schwedler domes is approximately identical for small or large areas. Thus, this type of domes can be considered as a good choice to cover large areas without intermediate columns.

Another study was also carried out by Kaveh and Talatahari [19] in 2010. In this paper the HBB-BC algorithm, previously proposed by the authors, was employed for optimal design of skeletal structures considering discrete variables. Four problems consisting of two trusses and two frames were considered to investigate the performance of the algorithm. It was demonstrated that the HBB-BC method locates optimum solutions which are better or in some cases well comparable with those obtained by other heuristic methods.

In 2010, Tang et al. [20] employed the BB-BC optimization method for parameter estimation of structural systems. In this optimization problem the aim is to find the best parametric values to minimize the error between an actual physical measured response of a structure and the simulated response of a mathematical model. The response of a multi-story shear-frame structure was considered for evaluation purposes. Numerical results revealed that the BB-BC method is able to obtain high quality solutions with better computational
efficiency compared to the GA and PSO methods.

The optimum design of 2D reinforced concrete frames via two metaheuristic methods (one of which was BB-BC) was investigated by Kaveh and Sabzi [21] in 2011. A variant of the BB-BC method which was previously developed by Camp [16] in 2007 was implemented. In addition, a harmony search scheme was used. To this end, each component of candidate solutions generated by the BB-BC algorithm, which violated the variables boundary, is regenerated using the harmony search method. Three frame structures with 4, 8, and 12 stories subjected to vertical and lateral loading were considered as numerical examples. The load combinations and design methodology was based on ACI-318-08 [22] code. Numerical results demonstrated the robustness of the BB-BC method and its capability to obtain optimal or near optimal designs. The simplicity of the implementation of the method was also emphasized in the study.

In 2011, Kaveh and Abbasgholiha [23] studied the optimum design of steel sway frames using the BB-BC algorithm. To this end, four 2D frames were designed based on AISC-LRFD-92 [24] and BS5950-90 [25] codes using the BB-BC variant developed by Camp [16]. Both of strength and displacement constraints were considered, with only one load combination. It was shown that the BB-BC method obtained more economical designs than the previous methods. In 2011, some researchers investigated the performance of BB-BC in solving engineering optimization problems. Kazemzadeh Azad et al. [15] evaluated the efficiency of the BB-BC algorithm in benchmark engineering optimization problems. Several well known benchmark instances such as the cantilever welded beam and pressure vessel problems were solved via BB-BC algorithm and the results were compared to those of other methods. The best, worst, mean, and standard deviation of the results were reported to provide an overview of the general performance. It was concluded that the BB-BC method is effective in solving complex engineering problems.

The optimum design of truss structures considering natural frequency constraints was studied by Kaveh and Zolghadr [26] in 2012. These problems are generally considered to be highly nonlinear and to have many local optima. A hybrid CSS-BBBC algorithm with a trap recognition mechanism was proposed. This mechanism senses the local optima and uses the big-bang scheme of BB-BC method to help agents leave the trap (i.e. local optimum). In addition, the developed method eliminated the parameter tuning phase of the CSS method. Several numerical examples were solved with the proposed method. The numerical comparisons revealed that the proposed new method is able to find better results than previous methods in all of the investigated examples.

An improved variant of the BB-BC method called the exponential BB-BC was proposed by Hasançebi and Kazemzadeh Azad [27] in 2012. The new EBB-BC method was proposed to improve the performance of the original BB-BC method, specially in discrete structural optimization problems. A simple, yet effective, modification was carried out in the original expression of the BB-BC method. It was suggested to increase the power of the random variable used in the Big-Bang phase to three, instead of unity. Also, the random variable can be a normally distributed (MBB-BC) or an exponentially distributed number (EBB-BC). The performances of the new approaches were investigated by considering the code-based design optimization of steel frames. It was demonstrated that the MBB-BC algorithm has better exploration characteristics than the standard BB-BC method while the EBB-BC is more successful in adjusting the balance between exploration and exploitation and performs
better than the others.
In 2012, Camp and Akin [28] employed the BB-BC method for optimum design of cantilever reinforced concrete retaining walls. Stability and strength constraints were considered and the design was based on ACI 318-05 [29]. Two design examples were solved and it was concluded that the BB-BC method can be used readily for optimum design of retaining walls considering its simplicity and efficiency. In the same year, Kaveh and Eftekhar [30] used the BB-BC variant developed by Kaveh and Talatahari [17] and hybridized it with HS for optimum design of double layer barrel vaults and reported satisfactory results.
Camp and Huq [31], in 2013, utilized a BB-BC algorithm for CO$_2$ and cost optimization of reinforced concrete frames. The hybrid BB-BC algorithm previously developed by Kaveh and Talatahari [17] was modified to include a two-phase search scheme. In the first phase the design space was searched extensively while in the second phase a relatively smaller region around the best solution (found in the first phase) was considered. Three 2D reinforced concrete frame design examples were solved. The BB-BC designs of frames showed some improvements (about 6% reduction in cost) over the designs developed by GA and SA. In another study, Camp and Assadollahi [32] used the above BB-BC algorithm in optimum design of reinforced concrete footings under vertical column loads. Geotechnical and structural constraints were considered based on ACI-318-11[33]. Several design examples were solved and the evaluations were based on either cost or CO$_2$ emission objective functions. It was demonstrated that the proposed variant of the BB-BC is both effective and able to generate low-cost and low-CO$_2$ emission spread footing designs. Two other studies considering reinforced concrete spread footings under uniaxial and biaxial uplift were also conducted by these authors later [34-35].
An Upper Bound Strategy (UBS) for optimum design of steel frames was proposed by Kazemzadeh Azad et al. [36] in 2013 which can be used in conjunction with many metaheuristic algorithms, specially the BB-BC method. The UBS states that after generation of a new population in the big-bang phase, it is not necessary to analyze all of these candidates. Instead, only a limited number of candidates with weights less than the best solution found so far shall be analyzed. Thus, if the net weight (not penalized weight which requires analysis) of a candidate is more than the penalized weight of the best solution found so far, it will not be analyzed. Two real-size 3D steel frames i.e. 135-member and 1026-member steel frames shown in Fig. 1 were considered under 10 load combinations as numerical examples and solved using the original BB-BC as well as its two improved variants namely MBB-BC and EBB-BC previously developed by the authors. The numerical results attained through design optimization of the above mentioned real size steel frames revealed that the UBS is capable of reducing the computational effort required to obtain a reasonable design. More specifically it was demonstrated that employing UBS in BB-BC algorithm can reduce the number of structural analysis up to 90% while retaining the main explorative and exploitative characteristics of the method. As a continuation to this study, Kazemzadeh et al. [37] conducted another research on the optimum design of trusses. It was demonstrated that the UBS algorithm can reduce the number of structural analysis up to 50% in the optimization process of steel truss structures via the BB-BC method.
In 2013, Tabrizian et al. [38] utilized a two-phased BB-BC algorithm to detect damage location and severity in structural systems. Through three numerical examples it was demonstrated that the BB-BC algorithm can effectively and accurately detect damage in structural members. Kaveh and Mahdavi [39] combined the mathematical algorithm of Quasi-Newton with the BB-BC variant developed previously by Kaveh and Talatahari [17] and used it in the optimal design of structures with multiple natural frequency constraints. In their approach first the BB-BC method performs a global search and next the Quasi-Newton method conducts a local search. The aim was to improve the local search capabilities of the BB-BC algorithm while retaining its explorative nature. Four truss examples and two frame examples were solved using the proposed algorithm and satisfactory results were reported. Saha [40] proposed a modified BB-BC and used it for locating the critical surface in slope stability theory. The stability of soil slopes was evaluated where the factor of safety against failure was considered as the objective function. It was demonstrated that the proposed BB-BC method can obtain better designs than other traditional and some metaheuristic methods. Talatahari et al. [41] modified the BB-BC variant developed previously by Kaveh and Talatahari [17] by introducing a two-stage search scheme and used it to estimate the optimal values for the parameters of the mathematical models of non-linear systems using the response obtained from experimental data. The objective was to minimize the error between the mathematical and experimental results. An example based on the Bouc-Wen hysteretic model of a sample damper was solved and it was reported that, although the original BB-BC could not find satisfactory results in some cases, the new proposed BB-BC variant was able to find results with a maximum error of 10% which was well-accepted for the investigated example.

In 2014, Hasançebi and Kazemzadeh Azad [42] employed the MBB-BC algorithm for discrete size optimization of steel truss structures. A total of four comprehensive design examples were solved to demonstrate the robustness and efficiency of the proposed algorithm. The effect of the power of the random variable used in the big-bang phase was also investigated and the value of 3 was reported to be suitable. In the same year, Kazemzadeh Azad et al. [43] used the EBB-BC algorithm along with the UBS to optimize two large-scale 3D steel frames shown in Fig. 2 with practical design constraints and loading conditions. One of these examples composed of 11540 members is amongst the largest steel
frame instances investigated so far in the literature. It was successfully demonstrated that the UBS integrated optimization techniques can be efficiently utilized for practical optimum design of large-scale structures using regular computers.

In 2015, Hadidi and Rafiee [44] proposed a new hybrid HS-BB-BC algorithm in which the merits of both HS and BB-BC methods were combined. The proposed method was employed in the size and semi-rigid connection type optimization of steel frames. Geometrical nonlinearity effects such as nonlinear moment-rotation behavior as well as the P-Δ effects were included in the structural analysis phase. The goal was to find the member sections and semi-rigid connection types to minimize total member plus connection cost. Stress, displacement, and beam-column size compatibility constraints were considered. In their study three benchmark frame examples were solved and it was reported that the HS-BB-BC algorithm outperformed the HS and BB-BC algorithms by obtaining lighter designs faster. It was also noted that the T-stub connection is an economical connection type for unbraced steel frames.

4. RUNTIME COMPARISON OF DIFFERENT BB-BC VARIANTS

Although for small and medium size structures having a few design variables the main part of computational effort of optimization process is usually devoted to the structural analysis stage, in case of large scale structures with numerous design variables computational time required for generation of new candidate solutions during the optimization process could be considerable. Therefore, having an approximate insight into the runtime of optimization algorithms could be fruitful. As mentioned in the previous section there are many variants of the BB-BC algorithm available in the literature of structural optimization. In this section an empirical comparison is performed between the runtime of three different variants of the BB-BC algorithm. The first variant is based on the standard version of the algorithm improved in Ref. [42] for discrete sizing optimization. The second version of the algorithm namely MBB-BC refers to the third power reformulation of the algorithm according to a normally distributed random number proposed for discrete optimization in Ref. [42].
third version of the algorithm, namely EBB-BC, the use of an exponential distribution in conjunction with the third power of random number is proposed Ref. [27]. It is worth mentioning that comparison of performance of these variants of the BB-BC in terms of solution quality can be found in Refs. [27, 42] and is not intended here. Instead runtime of each variant over a predetermined maximum number of iterations is considered.

In order to investigate the runtimes of the above mentioned variants, the 38-member planar truss structure shown in Fig. 3 is chosen. This structure is the test instance of the second International Student Competition in Structural Optimization (ISCO 2012) [45]. Here geometry and topology of the truss is assumed to be fixed, and only sizing optimization of the structure is considered. A vertical load of $P=15$ kips is applied to the structure at node 21. The stress limit is 30 ksi in both tension and compression for all the members; and displacement of all nodes in both horizontal and vertical directions is limited to $\pm 4$ in. The material density is 0.283 lb/in.$^3$ and the modulus of elasticity is 30,000 ksi. The cross-sectional areas of truss members are considered as 38 sizing variables which should be selected from the list $A = \{0.1, 0.2, 0.3, \ldots, 14.8, 14.9, 15\}$ in.$^2$. In this design example, the self weight of the structure is not considered as an applied load.

![Figure 3. 38-member test example of ISCSO (2012), $a = 100$ in](image)

Sizing optimization of the 38-member truss is carried out using the standard, modified and exponential variants of the BB-BC algorithm, and the best designs, over 10 independent runs, are tabulated in Table 1. For all the investigated versions of the BB-BC the population size is set to 50 individuals and the maximum number of iterations is taken as 500 iterations. The algorithms are coded in MATLAB [47] and the optimization runs are performed using a regular PC with Intel Core i7-4720HQ, 3.6 GHz CPU and 8 GB RAM. In order to compare the results, the best designs found and average value of 10 runtimes for each variant is reported in Table 1. Regarding the best design weight reported for this example i.e. 5889.99 lb by MunichOpt, the winner team of the ISCSO (2012) [46], it is observed that all the three variants of the BB-BC algorithm are able to locate a promising solution for this example. The convergence histories of the best runs of the BB-BC, MBB-BC, EBB-BC are depicted in Fig. 4. In terms of optimality of final solutions, more general performance evaluations of the three variants of the BB-BC through challenging discrete sizing optimization problems can be found in Refs. [27, 42]. In this example, in terms of runtime of each variant, it was observed that the average runtimes of the BB-BC, MBB-BC, and EBB-BC for 500 iterations were 14.54, 14.47, and 22.77 seconds, respectively. The considerably higher runtime of the EBB-BC, compared to the other two variants, can be attributed to the use of exponential distribution for generation of the new candidate solutions in the course of optimization. It is worthwhile to note that for a general conclusion on runtimes, comprehensive investigations are required through different parameter settings, test examples, algorithmic
implementations, programming languages, etc. The runtime comparison presented here is a preliminary evaluation that could be useful to further highlight the effect of algorithmic implementations of the BB-BC variants on total computational time of the optimization process.

Table 1: Runtimes and obtained designs of different BB-BC variants

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Weight (lb) 5889.99 5891.16 5891.16
Runtime (sec) 14.54 14.47 22.77

Figure 4. Convergence histories of the BB-BC variants
In the recent decades an extensive research has been carried out on developing efficient structural optimization techniques for practical applications. Generally, a majority of the developed techniques belong to the class of stochastic search algorithms or the so called metaheuristics. As a popular metaheuristic optimization technique, the BB-BC algorithm has been widely employed by the researchers for solving different types of structural optimization problems so far. In addition to the common features of metaheuristic algorithms, i.e. their independency on gradient information, inherent capability of dealing with both discrete and continuous design variables, and stochastic global search features to locate promising solutions for complicated problems, the simple algorithmic structure of the BB-BC plays an important role in popularity of this technique in structural optimization applications. In this review study some of the most recent applications of the BB-BC algorithm in structural optimization were summarized. Different variants of the algorithm as well as attempts to reduce the total computational effort of the algorithm in structural optimization problems were briefly discussed. Furthermore, an empirical comparison is performed between the runtimes of three different variants of the algorithm. It is observed that the exponential version of the BB-BC has a higher runtime compared to the other two versions of the algorithm investigated in this study. The recent applications of the BB-BC algorithm revealed that the improved variants of the technique can be efficiently employed for optimum design of large scale steel structures under numerous load combinations stipulated by the standard design codes.

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