WEAR RATE PREDICTION OF GRINDING MEDIA USING BPNN AND MLR MODELS IN GRINDING OF SULPHIDE ORES

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Abstract: Nowadays steel balls wear is a major problem in mineral processing industries and forms a significant part of the grinding cost. Different factors are effective on balls wear. It is needed to find models which are capable to estimate wear rate from these factors. In this paper a back propagation neural network (BPNN) and multiple linear regression (MLR) method have been used to predict wear rate of steel balls using some significant parameters including, pH, solid content, throughput of grinding circuit, speed of mill, charge weight of balls and grinding time. The comparison between the predicted wear rates and the measured data resulted in the correlation coefficients (R), 0.977 and 0.955 for training and test data using BPNN model. However, the R values were 0.936 and 0.969 for training and test data by MLR method. In addition, the average absolute percent relative error (AAPE) obtained 2.79 and 4.18 for train and test data in BPNN model, respectively. Finally, Analysis of the predictions shows that the BPNN and MLR methods could be used with good engineering accuracy to directly predict the wear rate of steel balls.

Keywords: Wear rate, Steel balls, Grinding, Neural Network, Multiple Linear Regression.

1. INTRODUCTION

Comminution is one of the most important unit operations in the mineral processing industry. It is the most energy intensive operation which constitutes the major portion of operating and capital costs of the mineral processing plants [1].

Traditionally, grinding mills were only considered as devices to reduce particle size and liberate minerals so that value metals can be extracted economically by flotation, hydrometallurgy, pyrometallurgy or a combination of them. In the last 10 years the importance of grinding mills in preparation of suitable surface chemistry for the downstream operation has been recognized [2]. Ball mills are the most common and versatile type of grinding mills. They are remarkable in that they can operate over a very wide range of conditions and geometries. Ball mills may be used for primary, secondary, tertiary and regrind applications [3]. Mild steel or stainless steel balls are generally used as grinding media in ball mills for processing of sulphide ores to achieve the required liberation size.

Consumption of grinding media forms a significant part of the operating cost of a mineral processing plant. It is estimated that comminution includes 30–50% of typical mining operating costs [4], and on the other hand, media wear can constitute up to 40–45% of the total cost of comminution [5].

The grinding balls loses is contributed by different forces such as abrasion, erosion, impact and corrosion [6]. Abrasion and impact wear is metal loss due to mechanical force on the grinding media. Erosion wear results from the friction between grinding media and particles. Corrosive wear is defined as metal loss due to chemical and/or electrochemical reactions of grinding media with the solution and/or other electrochemical conductive [7-8]. Individual contributions of the above towards overall ball wear cannot be accurately estimated. However, the corrosion component in a wet grinding mill can be estimated by establishing the differences in ball wear observed under nitrogen and oxygen aeration conditions [6 and 9].

Many factors affect the metal loss in milling operations such as the composition and metallurgical properties of the grinding media, mineral properties (particularly hardness and
particle size), properties of the pulp such as solid/liquid ratio and pH, and mill operating conditions [10]. Therefore, the effects of these variables should be fully determined to obtain acceptable grinding. A vast number of studies were carried out on the influence of these factors in wear of grinding media [6-18]. Classic approaches of studying the effects of operating variables on loses of the grinding balls in mills relies on the empirical models. Empirical models are obtained from statistical correlations between dependent and independent variables.

The main advantage of empirical models is its cost-effective way of using personnel and time. However, predictions outside the range of the model’s database should be treated warily. One of the problems with statistical empirical models is series of simplifications and multiplicity of the factors which should be taken into consideration in the balls wear process which complicates any modeling, so a full mathematical representation is difficult.

In recent years, artificial neural networks (ANN) have become a very useful and powerful tool in the modeling of the input-output relationships of complicated systems. It was applied successfully in many area including industrial processes, modeling the greenhouse effect, bleaching of metals, simulation N2O emissions from a temperate grassland ecosystem, prediction of materials properties such as steel, Prediction of the effect semisolid metal processing parameters, prediction of coal microbial, chemical desulphurization and operational parameters as well as coal Hargrove grindability index, modeling of the effect of cooling slope casting parameters on particle size of primary silicon crystals, gold content estimation in pyrometallurgy, estimation of coal swelling index based on chemical properties of coal, Prediction of heavy metals in acid mine drainage, and etc [19-36]. Furthermore, a large number of researchers reported application of neural networks models in predictions of wear loss [37-47].

The ANN utilizes interconnected mathematical nodes or neurons to form a network that can model complex functional relationships. The technique is particularly suited to problems that involve the manipulation of multiple parameters and non-linear interpolation, and as a consequence are therefore not easily amenable to conventional theoretical and mathematical approaches [48]. In fact, the main advantage of ANN is the ability to modeling a problem by the use of examples (i.e. data driven), rather than describing it analytically. Unlike multiple linear or nonlinear regression techniques, which require a predefined empirical model, neural networks can identify and learn the correlative patterns among the input and corresponding output values once a training data set is provided. Neural networks learning algorithms can be divided into two main groups that are supervised and unsupervised. In general, BPNN model seems to be the most utilized neural network for researchers in various process modeling applications.

As considered, the literature review indicates that ANN and MLR methods can be very good choice in this regard, as these models exhibit significant ability in estimating of output and simulating various process especially ANN method. Therefore, this study were aimed to estimate the loses of steel balls and to simulate of wear process using ANN (BPNN model) and MLR methods and finally, the result obtained from predictions of these two ways are compared with the measured wear rates of steel balls.

2. MATERIAL AND METHODS

2.1. Materials

The obtained samples from the ball mills input of the Sarcheshmeh copper mine were crushed in a jaw crusher (Fritsch 01.703). The size fraction of -2000 +250 micrometers was collected for experiments. Samples was then homogenized and sealed in polyethylene bags. Samples were chemically analyzed which contained 0.74% Cu, 4.34% Fe, 0.032% Mo, 3.05% S, 55.07% SiO<sub>2</sub> and 14.35% Al<sub>2</sub>O<sub>3</sub>.

Steel balls used in Sarcheshmeh concentrator plant were employed as grinding media, which their chemical compositions presented in Table 1. Samples were ground under different experimental conditions in a laboratory ball mill.
Table 1. Chemical compositions of the grinding media

<table>
<thead>
<tr>
<th>Ball type</th>
<th>Chemical compositions (Weight, %)</th>
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<tr>
<td>Low alloy steel (LS)</td>
<td>C</td>
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with 6-14 kg ball (mixing of 0.5, 0.75 and 1 inch ball in diameter) at grinding time, 7.5-17.5 minutes and rotation speed, 70-85 rpm such as 70% of particles were less than 75 μm in diameter. In determining the ball mass losses through total wear, 15 steel balls were handpicked and marked and then before and after each grinding experiment were weighted to calculate the ball losses. Finally, the wear rate in mils penetration per year was calculated from following Equation [49].

\[ CR = \frac{534 \times W}{\rho \times A \times t} \]  

(1)

where CR is the wear rate in mils penetration per year (mpy), W is weight loss in milligrams, ρ is density in grams per cubic centimeter, A is area in square inches, and t is time in hours.

2. 2. Back Propagation Neural Network (BPNN)

Artificial neural networks (ANN) are generally defined as information processing representation of the biological neural network. ANN is very powerful tools to effectively represent complex non-linear systems. It is also considered as a non-linear statistical identification technique. Figure 1 shows a typical neuron. Inputs (P) coming from another neuron are multiplied by their corresponding weights (W1, R), and summed up (n). An activation function (f) is then applied to the summation, and the output (a) of that neuron is now calculated and ready to be transferred to another neuron [50-52].

According to learning algorithms several types of neural networks such as Back propagation Neural Network (BPNN), Probabilistic Neural Network (PNN) and General Regression Neural Network (GRNN) have been designed in MATLAB software [50-52].

The feed-forward neural networks with back propagation (BP) learning algorithm are very powerful in function optimization modeling [53-54]. A BPNN model is one of the most commonly used neural network [55]. In BPNN model, the neurons are arranged in layers and are connected so that the neurons in a layer receive inputs from the preceding layer and sends out outputs to the

![Fig. 1. A typical neuron (Demuth and Beale, 2002)](image-url)
following layer. External inputs are applied at the 
first layer and system outputs are taken at the last 
layer. The intermediate layers are called hidden 
layers. A single hidden layer of the back-
propagation has been proven to be capable of 
providing accurate approximations to any 
continuous function provided there are sufficient 
hidden units [54]. The basic concepts of the back-
propagation algorithm can be found in the 
literature [56-58].

3. PREDICTION OF WEAR RATE BY USING 
MODELS

3.1. Predicting the Wear Rate of Steel Balls Using 
BPNN

In this study, 50 experimental datasets of 
grinding were used to create BPNN model. Table 
2 gives experimental conditions and calculated 
wear rate of steel balls. Input parameters of 
BPNN include pH, solid percentage (%), 
throughout (weight of samples input ball mill) 
g), charge weight of balls (kg), rotation speed of 
mill (rpm) and grinding time (minutes). The 
output of network is wear rate of steel balls in 
ils penetration per year.

Table 3 shows the correlation matrix between 
wear rate of steel balls and independent variables 
that effect on cuttings transport using SPSS 
software. According to this table, pH, solid 
percentage, grinding time and speed of mill are 
more effective on wear rate of steel balls.

The BPNN model was trained using 40 
randomly selected data (accounting for 80% of 
the total data) while the remaining 10 data 
(accounting for 20%) were utilized for testing 
purposes. Several architectures comprising 
varied numbers of neurons in hidden layer with 
ABR algorithm were tried to predict cutting 
concentration using BPNN. Considering the 
requirements of the ANN computation algorithm 
(better identification of parameters), both input 
and output data were normalised to an interval by 
a simple transformation process. In this study, 
normalization of data was carried out within the 
range of [-1, 1] using Equation (2) [52]:

\[ p_n = 2 \frac{p - P_{\text{min}}}{P_{\text{max}} - P_{\text{min}}} - 1 \]  

(2)

where, \( p_n \) is the normalised parameter, \( p \) denotes 
the actual parameter, \( P_{\text{min}} \) represents a minimum 
of the actual parameters and \( P_{\text{max}} \) stands for a 
maximum of the actual parameters.

In addition, two criteria were employed in 
order to assess the effectiveness of each network 
and its ability to make accurate predictions; they 
are: average absolute percent relative error 
(AAPE) and the correlation coefficient (R) [59].

The AAPE concept gives an idea of absolute 
relative deviation of estimated from the measured 
data. It can be calculated from the following 
equation:

\[ AAPE = 100 \times \frac{1}{N} \sum_{i=1}^{N} \left| \frac{y_i - \hat{y}_i}{y_i} \right| \]  

(3)

where, \( y_i \) is the measured value, \( \hat{y}_i \) denotes the 
predicted value, and \( N \) stands for the number of 
samples. The lowest AAPE value, the more 
accurate the prediction is.

The last measure, known as the efficiency 
criterion, \( R \) represents the percentage of the 
initial uncertainty explained by the model. It is 
given by:

\[ R = \sqrt{1 - \frac{\sum_{i=1}^{N} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{N} y_i^2 - \frac{\sum_{i=1}^{N} \hat{y}_i^2}{N}}} \]  

(4)

The best fitting between predicted and 
measured values, which is unlikely to occur, 
would have RMS=0 or \( R=1 \). The optimal 
network of this study is a feed forward multilayer 
perceptron [53-54] with Automated Bayesian 
Regularization (ABR) training algorithm for 
avoiding of over fitting problem in BPNN. This 
network comprises one input layer with 6 inputs 
(pH, solid percentage, throughout, charge weight, 
速度 and grinding time) and one hidden layer.
with 10 neurons. Fletcher and Goss (1993) suggested that the appropriate number of nodes in a hidden layer varies between \((m + n)\) and \((2n + 1)\), where \(n\) is the number of input nodes and \(m\) represents the number of output nodes) [60]. Each neuron has a bias and is fully connected to all inputs and employs a log-sigmoid activation function. The output layer has one neuron (wear

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<th>Charge weight (kg)</th>
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Table 3. Correlation matrix between cuttings concentration and independent variables

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<th>Throughput</th>
<th>Charge weight</th>
<th>Speed</th>
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<td>-0.204</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Speed</td>
<td>0.069</td>
<td>0.000</td>
<td>0.057</td>
<td>-0.276</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Grinding time</td>
<td>-0.069</td>
<td>0.000</td>
<td>-0.057</td>
<td>0.141</td>
<td>0.139</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Wear rate</td>
<td>-0.531</td>
<td>-0.530</td>
<td>-0.195</td>
<td>-0.022</td>
<td>0.267</td>
<td>0.435</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 2. BPNN architecture (6-10-1)

rate) with a linear activation function (purelin) without bias. Training function of this network is ABR algorithm (trainbr). In this study, (n=6) and (m=1) and thus the appropriate number of hidden layer neurons was chosen as 10 (6-10-1).

Schematic description of the constructed BPNN architecture is shown in Figure 2.

3. 2. Predicting Wear Rate of Steel Balls Using MLR

Multiple linear regression (MLR) is an extension of the regression analysis that incorporates additional independent variables in the predictive equation. Mathematically [59]:

\[ y = B_1 + B_2x_2 + \ldots + B_nx_n + e \]  \hspace{1cm} (5)

where, \( y \) is the dependent variable (wear rate), \( x_1 \) is the independent random variables and \( e \) is a random error (or residual) which is the amount of variation in \( y \) not accounted for by the linear relationship. The parameters \( B_i \), stand for the regression coefficients, are unknown and are to be estimated. However, there is usually substantial variation of the observed points around the fitted regression line. The deviation of a particular point from the regression line (its predicted value) is called the residual value. The
smaller the variability of the residual values around the regression line, the better is model prediction. In this study, regression analysis was performed using the train and test data employed in neural network data. The wear rate of steel balls considered as the dependent variable. A computer-based package called SPSS (Statistical Package for the Social Sciences) was used to carry out the regression analysis.

4. RESULTS AND DISCUSSION

4.1. BPNN Results

Using the BPNN method described above, all necessary computations were implemented by supplying extra codes in MATLAB software. The matrix of inputs in training step is a \( n \times N \) vector, where \( n \) is the number of network inputs and \( N \) is the number of samples used in training step. In this work, six input variables (pH, solid percentage, throughput, charge weight, speed and grinding time) and 40 samples were used to train the network; therefore \( n \times N = 6 \times 40 \). The matrix of outputs in training step is a \( m \times N \) vector, where \( m \) is the number of outputs. In this study, there is only one output so, \( m \times N = 1 \times 40 \). In the same manner, the matrices of inputs and output for testing phase, were \( n \times N = 6 \times 10 \) and \( m \times N = 1 \times 10 \) respectively. The correlation coefficient (R) and AAPE were used in order to evaluate the effectiveness of ANN model and its ability to make accurate predictions. Figure 3 compares the predicted wear rate of steel balls and the measured values for the training data set. The correlation coefficient (R) to the linear fit (\( y=ax \)) is 0.977 with the AAPE value of 2.79%, describing almost a perfect fit. The very good fitting values indicate that the training was done very well.

For testing stage, those data sets which were not employed by the ANN model during training process were used. Figure 4 provides the results of the comparison between the wear rates of steel balls from the experimental and estimated results in the testing set. The correlation coefficient (R) and AAPE between the estimated and experimental values obtain 0.955 and 4.18, respectively. These results verified the success of neural networks which recognize the implicit relationships between input and output variables.

4.2. MLR Results

The purpose of regression analysis is to determine the values of parameters for a function that causes the function to best fit a set of data provided. Multiple regressions analysis solves
the data sets by performing least squares fit. Using MLR approach in SPSS software, the estimated regression relationship for wear rate of steel balls is given as following:

\[
CR = 616.221 - 41.553 A - 7.994 B - 0.044 C + 7.466 D + 4.504 E + 8.521 F
\]  

(6)

where CR, A, B, C, D, E and F represent the wear rate of balls (mpy), pH, solid content (%), throughput (g), charge weight (kg), speed (rpm) and grinding time (min), respectively.

The statistical results of the model are given in Table 4.

Wear rate of balls was estimated according to the Equation 6. Figures 5 and 6 compare the MLR wear rates versus the experimental values for the training and test data set respectively. The correlation coefficient (R) and AAPE for train data are 0.936 and 4.3% and for test data, they are 0.969 and 4.07%.

A comparison of the ANN and MLR predictions with the measured values for the all data sets used in this study with a population of 50 is shown in Figure 7. The correlation coefficient (R) is 0.969 and 0.942 for ANN and MLR model respectively. The AAPE values are 3.07 % and 4.25% for ANN and MLR model, respectively.

**Table 4.** Statistical characteristics of the multiple regression model

<table>
<thead>
<tr>
<th>Model</th>
<th>Method</th>
<th>Independent variables</th>
<th>Coefficient</th>
<th>Standard error</th>
<th>Standard error of estimate</th>
<th>t-value</th>
<th>F-ratio</th>
<th>Sig. level</th>
<th>Correlation coefficient (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eq.6</td>
<td>Enter</td>
<td>(Constant) 616.221 86.114</td>
<td>7.156</td>
<td>.000</td>
<td>23.414</td>
<td>39.414</td>
<td>.000</td>
<td>0.936</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>pH -41.553 4.339</td>
<td>-9.576</td>
<td>.000</td>
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<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Solid percentage -7.994 845</td>
<td>-9.462</td>
<td>.000</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Throughput -0.044 0.024</td>
<td>-1.847</td>
<td>.074</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Charge weight 7.466 2.515</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Speed 4.504 895</td>
<td>5.033</td>
<td>.000</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Grinding time 8.521 1.730</td>
<td>4.926</td>
<td>.000</td>
<td></td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>

![Fig. 5. MLR prediction versus measured wear rate (mpy) for the train data](image1)

![Fig. 6. MLR prediction versus measured wear rate (mpy) for the test data](image2)
These results demonstrate that the output, i.e. the estimated wear rates, matched the experimental data very well, indicating that the proposed BPNN and MLR models are capable of successfully predicting wear of steel balls under different experimental conditions.

5. CONCLUSION

In this study, wear rate of steel balls within the grinding of sarcheshmeh copper sulphide ore was estimated using artificial neural network (BPNN model) and MLR method. The ANN presented here has three layers namely input layer, hidden layer and output layer. Input layer has six neurons including, pH, solid percentage, throughout of grinding circuit, rotation speed mill, charge weight of balls and grinding time. Hidden layer has ten neurons with a log-sigmoid activation function in all neurons. Output layer has one neuron (wear rate) with a purelin activation function. The correlation coefficient between measured and prediction values in training and testing data in ANN model is 0.977 and 0.955, respectively. The AAPE of training and testing data in ANN model are 2.79% and 4.18% respectively. A comparison of the ANN and MLR for all data was done. The correlation coefficient (R) is 0.969 and 0.942 for ANN and MLR model respectively. The AAPE values are 3.07 % and 4.25% for ANN and MLR model, respectively. The results obtained from this study reveal that both ANN and MLR models could accurately predict the wear rate of steel balls, but the accuracy of ANN model is better respect to MLR model.

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